Application of particle swarm optimization theory in skew distribution of propeller

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**Abstract:** In order to reduce the excitation force of the propeller on a ship's hull surface, a hybrid method of the Particle Swarm Optimization (PSO) algorithm of the intelligent optimized field and panel method is adopted to optimize the propeller's skew distribution. The mathematical model and main process of the PSO algorithm in the optimization design of the skew distribution is given, and a Seiun-Maru HSP propeller is taken as the prototype. Three different skew distributions are obtained through calculations. The results show that the axial coefficients of thrust and torque for the optimal scheme are reduced significantly at Blade Passing Frequency (BPF) and 2 BPF, while the thrust and torque of the propeller show no such reduction. In brief, optimizing the skew distribution of the blades effectively improves the propeller's performance in unsteady fields, and the PSO algorithm is proven to optimize skew distribution for propeller engineering.

**Key words:** skewed propeller; panel method; particle swarm optimization; optimization design

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0 Introduction

When the propeller of ship rotates a round in the non-uniform three-way flow field\(^{[1-2]}\) behind the ship, the incoming-flow attack angle from each radius section of blade varies at any time with the changes of the angular wake, resulting in that the propeller undergoes periodic propeller excitation force. At present, skewed propeller is usually applied to reduce the inducing vibration of the propeller on the hull. The reasonable skew distribution form of the propeller makes the blade section at each radius of the propeller into the strong wake area\(^{[3-5]}\) in turn, and the distribution of the displacement volume of each blade along the circumference is more uniform, which effectively reduces the excitation force generated from the movement of the propeller in the non-uniform flow field\(^{[6]}\) and decreases the stern vibration and radiated noise.

In the past, the choice of skew distribution of propeller is often based on the empirical formula or refers to the prototype, where there are great limitations. In the engineering applications, the harmonic components\(^{[7-8]}\) of wake behind the ship must be considered in choosing the skew distribution, and different wake fields need to cooperate with the appropriate skew distribution form. Improper cooperation will have the opposite effect. In the preliminary design stage of propeller, through the harmonic analysis for the wake field behind the ship, the skew distribution form of propeller will be selected initially, and then the final skew distribution is determined through several manual adjustments based on the design experience. This artificial test design method is very tedious, and requires experienced designers, while this method cannot determine whether the final skew distribution form can achieve the best damping effect in the nonuniform flow field or not.

In this paper, the particle swarm optimization (PSO) algorithm is used, combined with the predic-
PSO algorithm

PSO algorithm is applied to search the optimal solution in complex space based on the collaboration and competition between individuals\textsuperscript{[9]}. The PSO algorithm randomly initializes particle swarm in the feasible solution space, and then determines the fitness value for each particle with the objective function. Each particle moves in the solution space and determines its direction and distance according to a speed. Usually, the particles will follow the current optimal particle to move, and finally get the optimal solution after generational search. In the iterative process, the particles will track two extreme values, which are the optimal solutions, pbest and gbest, respectively found by the particle itself and the population by far.

The main calculation steps of PSO algorithm\textsuperscript{[9-11]} are as follows:

1) Initialization. The acceleration constants, \( c_1 \) and \( c_2 \), and the maximum evolutionary generation \( T_{\text{max}} \) are set. The current evolutionary generation is set as \( t = 1 \). \( m \) particles \( x_1, x_2, \ldots, x_m \) are randomly generated in the declaration space \( \mathbb{R}^n \) to form the initial population \( X(t) \), and the initial displacement variations of each particle, \( v_1, v_2, \ldots, v_m \), are generated to form the displacement variation matrix \( V(t) \).

2) Evaluation for the population \( X(t) \). The fitness value of each particle is calculated in each dimensional space.

3) The comparison between the particle’s fitness value and its own optimal value pbest. If the current value is better than pbest, the current value is set as pbest and the pbest position is set as the current position in \( n \)-dimensional space.

4) Comparison between the particle’s fitness value and the population’s optimal value gbest. If the current value is better than gbest, the current value is set as gbest and the matrix subscript and fitness value of the current particle are set as gbest.

5) According to Eq. (1) and Eq. (2), the displacement direction and step size of the particles are updated to produce a new population \( X(t+1) \).

\[
\begin{align*}
X_{id}^{(t+1)} &= X_{id}^{(t)} + v_{id}^{(t+1)} \\
v_{id}^{(t+1)} &= v_{id}^{(t)} + c_1 r_1 \left( P_{id}^{(t)} - X_{id}^{(t)} \right) + c_2 r_2 \left( G_{id}^{(t)} - X_{id}^{(t)} \right) 
\end{align*}
\]  

where \( d = 1, 2, \ldots, n \); \( i = 1, 2, \ldots, m \); \( m \) is the population size; \( t \) is the current evolutionary generation; \( r_1 \) and \( r_2 \) are random numbers distributed in the interval \([0, 1]\); and \( c_1 \) and \( c_2 \) are the acceleration constants. In Eq. (1), the first item on the right is the particle’s previous speed; the second term is the cognition part, which indicates the particle’s own thinking; and the third term is the social part, indicating the information sharing and mutual cooperation between the particles.

6) Examination for the end condition. If the condition is met, the search is finished; otherwise, \( t = t + 1 \), and the research goes to step 2). The end condition is that the maximum evolutionary generation \( T_{\text{max}} \) is reached through optimizing, or the evaluation value is less than the given precision \( \varepsilon \).

2 Mathematical expression of the skew distribution of propeller

In 1962, Bezier, an engineer at Renault Company, France, proposed the Bezier curve model for the design of automotive components. Various shapes are directly designed on the computer based on the Bezier curve model. At present, this model has become one of the basic models of CAD software, and has been widely used in machinery, automotive design, font design and other fields.

Unlike the traditional interpolation spline curve, the Bezier curve is an approximation fitting curve\textsuperscript{[2]}, and the shape of the curve is visualized and relatively easy to be controlled. The shape of a Bezier curve is determined by the control polygon. In this paper, the Bezier curve is used to fit the skew distribution curve of propeller, and the reconstructions of crown distribution and thickness distribution are realized by changing the position of the control points to construct the new sample. The Bezier polynomial usually serves as the basis function of the Bezier curve, and an \( n \)-order Bezier curve is described as follows.

\[
p_u = \sum_{i=0}^{n} B_{i,n}(u) b_i, \quad 0 \leq u \leq 1
\]

where \( b_i (i = 0, 1, 2, \ldots, n) \) is the vertex of the characteristic polygon and \( B_{i,n}(u) (i = 0, 1, 2, \ldots, n) \) is the Bezier polynomial.

Bezier curve is featured with good convex hull property, and the fitting curve can be completely enveloped in the convex hull formed by the characteris-
tic polygon. The trend of the Bezier curve is determined by the shape of the characteristic polygon and the number of the inflection points on the curve can be determined by selecting the number of the control points. Therefore, the shape of the Bezier curve can be controlled by the order of the Bezier curve, which is very favorable for fitting the radial parameter distribution of the propeller. Fitting the radial parameter distribution of the propeller with the lower order Bezier curve can adequately guarantee the rationality of the radial parameter distribution of the optimized propeller.

The Bezier curve shape is only related to the position of the control point $V_i$. Fig. 1 shows a cubic Bezier curve and its control polygon. As shown in the figure, $p_0$, $p_1$, $p_2$, and $p_3$ represent the control points of the curve. It is possible to obtain a larger design space for the propeller with less change of the control point positions, and to meet the fairness requirements. In this paper, the cubic Bezier curve is used to fit the skew distribution form of the Seiun–Maru HSP propeller[13-15], the skew distribution curves before and after fitting are shown in Fig. 2. In the figure, $r/R$ is the relative radius, where $r$ is the radius of each section and $R$ is the propeller radius.

3 Optimization model

In this paper, the Seiun–Maru HSP propeller is served as the prototype, whose main parameters and the wake distribution are shown in Reference[16]. The prediction program of panel method for unsteady hydrodynamic performance of propeller and the Fourier's analysis method are used for the calculation. The fluctuation amplitude of the bearing force is used to evaluate the superiority of the skew distribution of propeller. In the case of ensuring propeller thrust, aiming to reduce the axial unsteady force and torque of the propeller at 1 blade passing frequency (BPF) and 2 BPF, the condition of minimizing the objective function is satisfied by adjusting the skew distribution to optimize the skew distribution of the propeller. Fig. 3 shows the comparison between surface pressure distribution and experimental values at the point of 0.7$R$ when the main blade of the propeller is at 0°, 90°, 180° and 270° during the rotation for one cycle[17]. In the figure, the ordinate $C_p$ is the pressure coefficient; $x/c$ is the relative chord length, where $x$ is distance from the point on the blade section along the chord direction to the guide edge, and $c$ is the chord length on the blade section. Fig. 4 shows the variations of the axial thrust coefficient and torque coefficient of the main blade during the rotation for one cycle, as well as their comparison with the Hoshino calculation results[18]. As seen in the figure, prediction program of panel method for unsteady hydrodynamic performance of propeller has high calculation accuracy and good stability, which can be used in propeller optimization design.
The mathematical model for skew optimization is described as follows:

Optimization target: \( \text{Min } \sigma_i \)

Limited condition: \( \sigma_i = \frac{|K_T - K_{T0}|}{K_{T0}} \leq \varepsilon_T \)

Optimization variable: \( S_L \leq S_i \leq S_U \)

Objective function: \( \sigma_i = \sum \omega_i K_i \)

where \( i \) is the blade frequency multiple; \( \omega_i \) is the weight coefficient of the function; \( K_i \) is the axial unsteady force and torque at the blade frequency and BPF of propeller. In this paper, for simplifying the calculation, only the axial force and torque at 1 BPF and 2 BPF are given. In the limited condition, \( \sigma_i \) denotes the error of the propeller thrust before and after optimization; \( \varepsilon_T \) is the acceptable thrust loss limit; \( K_{T0} \) is the thrust coefficient of original propeller; and \( K_T \) is the thrust coefficient after optimization. Through this limited condition, under the premise that the propeller thrust loss is within the acceptable range of designer, the optimal scheme that the objective function is minimum is obtained. The optimization variable \( S_i \) represents the skewed angle at each radius of the propeller, whose upper and lower limits are controlled by \( S_L \) and \( S_U \) so that the optimal scheme is in a reasonable skew range.

In this paper, the Bezier curve is used to fit the skew distribution curve of the propeller. The optimization variable \( S_i \) represents the position of the control points of the Bezier curve. In the geometric reconstruction of propeller, to ensure that the skew distribution form of propeller meets the actual engineering use requirements, the skew value cannot be too large. Therefore, this study gives the same range of values for the skewed angles of the different radius sections, that is, the skewed angle of each radius section is within 20% of the initial value of each skewed angle. The change range of the optimization variable is large, which can ensure that it contains the optimal scheme in line with the actual engineering requirements.

The first three order unsteady forces and torques exert main influences on the unsteady hydrodynamic performance of propeller. The magnitude of the unsteady force and torque with higher order is very small and negligible. Since the first and second order thrust coefficient and the torque coefficient are the main factors affecting the performance of the propeller, in which the first order fluctuation amplitude of the bearing force and moment account for the main part, during the research, the multi-objective optimization is carried out with the first and second order axial thrust coefficients \( K_{TX1} \) and \( K_{TX2} \) and the torque coefficients \( K_{QX1} \) and \( K_{QX2} \) as the optimization target. The weights of \( K_{TX1} \) and \( K_{QX1} \) are set as 1.0, and the weights of \( K_{TX2} \) and \( K_{QX2} \) are set as 0.5.

### 4 Analysis of the optimization results

In this paper, the multi-objective global optimization is carried out on the skew distribution of the Seiun-Maru HSP propeller with the PSO algorithm.
Three representative schemes are selected to analyze from the final optimization results. In the selected three schemes, Scheme A is the optimal scheme for the comprehensive effect; Scheme B is the optimal scheme for the fluctuation amplitude at 1 BPF; and Scheme C is the optimal scheme for the fluctuation amplitude at 2 BPF. The control points of the optimal schemes are shown in Table 1 and the skew distribution forms of the propeller before and after optimization are shown in Fig. 5.

The optimization results are summarized in Table 2. The results show that the fluctuation amplitude of force at 1 BPF has the same variation trend as that of torque, that is, they increase and decrease at the same time. The fluctuation amplitudes of force and torque at 2 BPF have the same law, too.

The axial thrust coefficient and torque coefficient of each scheme are shown in Fig. 6 and Fig. 7. It can be seen from the figures that the average value of the axial thrust coefficient remains almost unchanged, and the highest and lowest points are close to the center, which shows that the first-order fluctuation amplitude is significantly reduced. The average values of the axial thrust coefficient and torque coefficient before and after optimization are shown in Fig. 8 and Fig. 9. As can be seen in the figures, the changes of the axial thrust coefficient and torque coefficient before and after optimization are very small.

The comparison between the axial first-order blade–frequency thrust coefficient and torque coefficient of each scheme before and after optimization is shown in Fig. 10 and Fig. 11. Compared with the prototype, it can be seen from the figures that the axial thrust coefficient and torque coefficient at 1 BPF in Scheme A and Scheme B have a significant decrease, but in Scheme C, those at 1 BPF have a great increase. The comparison between the axial second order thrust coefficient and torque coefficient at 1 BPF in each scheme before and after optimization is shown in Fig. 12 and Fig. 13. As shown in the figures, comparing with the prototype, the thrust coeffi-

### Table 1 Coordinate of the control points

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Coordinate of the control points</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$(0.197 , 2.0),, (0.493 , 4,-0.241 , 8)$, $ (0.701 , 4,0.192 , 7)$, $(1.0,0.726 , 4)$</td>
</tr>
<tr>
<td>B</td>
<td>$(0.197 , 2.0),, (0.553 , 2,-0.218 , 8)$, $(0.732 , 3,0.120 , 9)$, $(1.0,0.726 , 4)$</td>
</tr>
<tr>
<td>C</td>
<td>$(0.197 , 2.0),, (0.530 , 1,-0.161 , 2)$, $(0.577 , 4,0.272 , 1)$, $(1.0,0.726 , 4)$</td>
</tr>
</tbody>
</table>

### Table 2 Optimization results summary

<table>
<thead>
<tr>
<th>Item</th>
<th>$K_{TX}$</th>
<th>$K_{QX}$</th>
<th>$1000 , K_{TX1}$</th>
<th>$1000 , K_{QX1}$</th>
<th>$1000 , K_{TX2}$</th>
<th>$1000 , K_{QX2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original propeller</td>
<td>0.171 4</td>
<td>0.028 5</td>
<td>4.790 9</td>
<td>0.776 2</td>
<td>0.189 7</td>
<td>0.043 6</td>
</tr>
<tr>
<td>Scheme A</td>
<td>0.170 9</td>
<td>0.028 4</td>
<td>4.575 4</td>
<td>0.729 8</td>
<td>0.178 3</td>
<td>0.039 5</td>
</tr>
<tr>
<td>Scheme B</td>
<td>0.170 1</td>
<td>0.028 3</td>
<td>4.477 1</td>
<td>0.721 8</td>
<td>0.187 8</td>
<td>0.042 3</td>
</tr>
<tr>
<td>Scheme C</td>
<td>0.171 3</td>
<td>0.028 5</td>
<td>5.270 0</td>
<td>0.846 0</td>
<td>0.165 8</td>
<td>0.038 7</td>
</tr>
</tbody>
</table>

![Fig.5 Comparison of the skew distribution before and after optimization](image)

![Fig.6 Comparison of $K_{TX}$ before and after optimization](image)
cient and torque coefficient at 2 BPF in Scheme A decrease significantly, and those in Scheme B are also lower with a smaller decrease. The decrease amplitudes of the axial thrust coefficient and torque coefficient at 2 BPF in Scheme C are the maximum.

5 Conclusions

In this paper, the PSO algorithm, combined with the prediction program of panel method for the unsteady hydrodynamic performance of propeller and Fourier harmonic analysis method, is applied to carry out the multi-objective optimization design of the skew distribution of propeller. The optimization results show that under the condition that the propeller thrust has no loss, the axial unsteady force and torque are reduced significantly at 1 BPF and 2 BPF after optimization. Through analysis, in the optimized feasibility schemes, the axial unsteady force and torque at 1 BPF have a decrease, but those at 2 BPF have an increase. When the optimization design of the skew distribution using PSO algorithm is car-
ried out, the operating conditions of the propeller need to be fully considered, and reasonable weight coefficients and control point coordinates should be set. This paper provides a new technological approach for the optimization of skew distribution of propeller and verifies the feasibility of this method, achieving the engineering of the optimization design of the skew distribution of propeller.

References


基于粒子群算法的螺旋桨侧斜分布优化

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摘 要：为了降低螺旋桨激振力，减小螺旋桨对船体的诱导振动，采用粒子群优化算法，结合螺旋桨非定常面元法预报程序及对螺旋桨的侧斜分布进行优化设计。给出侧斜分布的数学表达形式以及粒子群优化算法的数学模型，并以Scim—Maru HSP螺旋桨为母型桨进行优化设计，得到3种优化方案，其中，最优方案在不损失螺旋桨推力和扭矩的情况下，轴向一倍叶频、二倍叶频推力系数和扭矩系数明显降低，达到了优化目的，即通过改变螺旋桨侧斜分布形式，能够有效改善非均匀流场中螺旋桨的性能，验证了粒子群优化算法用于螺旋桨侧斜分布优化的可行性，可实现工程化应用。

关键词：侧斜螺旋桨；面元法；粒子群优化算法；优化设计