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Automatic collision avoidance algorithm for unmanned surface vessel based on improved Bi-RRT algorithm



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Abstract: [Objectives] This method is proposed for automatic collision avoidance during high-speed running of the Unmanned Surface Vessel (USV). [Methods] The Bi-RRT algorithm was combined with the velocity obstacle algorithm for automatic collision avoidance algorithm for USV based on improved Bi-RRT algorithm. In view of the situation of the extension direction of the parent node inside the collision cone in the extended operation of Bi-RRT algorithm, the 'collision risk index' and 'obstacle repellent vector' were presented, making the extension direction of the parent node tend to move away from the obstacle. In addition, in view of the real time problem of the algorithm, the strategy of parallelly extending two search trees and the 'target attraction vector' when the extension direction of the parent node is outside the collision cone were introduced to accelerate the convergence of the algorithm. [Results] The results show that the algorithm using above-mentioned improvement method has reduced failed extension times of search tree, and the planned collision-free paths are shorter and smoother. [Conclusions] The research shows that the improved Bi-RRT algorithm has the advantages of high real-time performance and high path planning quality, which are of great significance for practical engineering application.

Key words: Unmanned Surface Vessel (USV); automatic collision avoidance; Bi-RRT algorithm; velocity obstacle algorithm

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0 Introduction

The unmanned surface vessel (USV) has the characteristics of high autonomy degree, fast navigation speed, good stealth performance, and strong maneuverability, and it is widely applied in such fields as hydrological detection, maritime cruise, and military operation. In recent years, with the resurgence of artificial intelligence technology and the large-scale application of automatic driving technology, as the blending outcome of intelligent technology and traditional ship discipline, the USV has become one of

the research hotspots of intelligent equipment all over the world.

The automatic collision avoidance is one of the key technologies for USV to realize intelligent navigation, which reflects the intelligence level of USV to some extent^[1-2]. Many scholars from various countries have also made some research achievements in USV collision avoidance. Svec et al.^[3] proposed three methods to prevent collisions with obstacles considering the impact of ocean wave on USV hull. The first method is to consider the boundary of obstacles for path planning. The second method is to con-

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sider the influence of external environment for revising the planned path and replanning. The third method is to combine A* heuristic search and locally bounded optimal planning under motion uncertainty using a variation of the minimax game-tree search. Kuwata et al.^[4] designed the USV collision avoidance method based on the velocity obstacle method and international rules for maritime collision avoidance. In the process of collision avoidance, the method set the minimum steering angle according to the maneuvering motion characteristics of USV and considered the influence of wind wave. Benjamin et al.^[5] integrated multiple thoughts such as behavior-based control framework principles and interval programming into a multi-objective interval optimization method to solve the collision avoidance problem of USV. Zhuang et al.^[6] designed a dynamic obstacle avoidance method based on the relative coordinate considering the international regulations and divided the USV collision into three different behaviors, i.e., overtaking, crossing, and head-on. Moreover, they built the USV semi-physical simulation platform. By analyzing the angular relationship between the relative velocity and relative position between USV and obstacle, Wu et al.^[7] proposed an automatic collision avoidance algorithm with the maneuvering motion characteristics of USV based on velocity obstacle method and considering the USV maneuvering characteristics under the influence of wind, wave and flow. Zhang et al.^[8] proposed a dynamic obstacle avoidance algorithm for USV based on velocity obstacle and dynamic window methods.

From the above research, the main idea of solving the collision avoidance problem of USV can be divided into two categories. One is to transform it into a local path planning problem and combine it with the intelligence algorithm. The other is to obtain the feasible range of collision avoidance path based on velocity obstacle method. At present, the research results combining them are relatively rare.

Inspired by previous research, this paper intends to combine the basic idea of velocity obstacle method with the Bi-RRT algorithm that has been successfully applied to Kinodynamic path planning problem^[9]. Firstly, the collision avoidance risk coefficient ω and the obstacle repulsion vector $\mathbf{R}$ are proposed for the situation where the extension direction of the parent node is located in the cone collision area, so that the search tree will extend towards the trend of moving away from the obstacle. Then, a method of parallel extension of two search trees and

the target attraction vector $\mathbf{A}$ triggered when the extension direction of the parent node is outside the cone collision area are proposed so as to improve the real-time performance of the algorithm and strengthen the trend of two search trees to approach the target point. Finally, the feasibility and superiority of the improved algorithm are proved by the simulation experiment.

1 Collision avoidance model of USV

In the process of autonomous navigation, the USV uses the intelligent sensing system to detect the scheduled route and the surrounding obstacle in real time and automatically avoid the obstacle. The collision avoidance problem of USV set in this paper is shown in Fig. 1. The specific description is as follows. At time t_0 , the vessel A is sailing along the scheduled route OS at speed V . The USV intelligent sensing system senses the route OS and the surrounding obstacle areas B_1 , B_2 and B_3 that have not been detected before. A circle with the overall length of vessel A as the diameter is made, and the area covered is regarded as the USV area. Based on the above situation, this paper designs an automatic collision avoidance algorithm based on the improved Bi-RRT algorithm, so that the vessel A can keep the original route as far as possible and avoid B_1 safely and efficiently under the limitation of B_2 and B_3 .

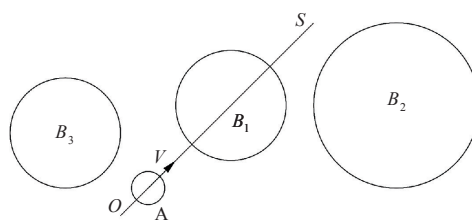


Fig.1 The situation of USV collision avoidance

2 Velocity obstacle method and Bi-RRT algorithm

2.1 Principle of velocity obstacle method

The velocity obstacle method was proposed by Fiorini et al.^[10]. The collision avoidance principle of this method is as follows. Firstly, based on the Featherstone theory, any polygon is represented as a corresponding series of circles. Meanwhile, in order to facilitate modeling and calculation, the USV and obstacle are replaced by different two-dimensional circles respectively. Then, the USV dimension is attached to

the obstacle dimension to calculate the velocity obstacle relation between them so as to simplify the USV into a particle and make the obstacle area expand into a circular area with a larger radius. Finally, the cone collision area is defined as the set of the relative velocity of the collision between USV and obstacle, which is geometrically represented as the two-dimensional area sandwiched by two tangents with USV as the vertex to the circumference of the obstacle. The relative velocity in the area outside the cone collision area will make the safe collision avoidance of USV.

The velocity obstacle method provides a simple and efficient basis for how to change and control the course during the collision avoidance process of USV and a theoretical basis for the application and improvement of Bi-RRT algorithm in the collision avoidance of USV.

2.2 Bi-RRT algorithm

The Bi-RRT algorithm was proposed by La Valle et al.^[11] inspired by bidirectional and heuristic search technology^[12]. Compared with the classic rapidly-exploring random tree (RRT) algorithm proposed by La Valle in 1998^[12], the Bi-RRT algorithm takes the starting point X_{init} and the ending point X_{goal} as the root node to construct two RRTs. The two trees grow together until they meet.

The Bi-RRT algorithm maintains its superiority because it keeps the idea of the classic RRT algorithm for path planning in the state space. La Valle^[13] tested the RRT algorithm for path planning under the conditions of holonomic constraint, nonholonomic constraint, Kinodynamic and closed motion chain, respectively. The results show that the algorithm is very efficient in the path planning of system with differential constraint and system with single-query nonholonomic constraint. The Bi-RRT algorithm is an improvement of the RRT algorithm. The termination condition of the algorithm is that the search tree growing with the starting point of path planning as the root node meets the search tree growing with the ending point of path planning as the root node, which can theoretically improve the search efficiency of position space. Meanwhile, in this algorithm, the operation of the extended nodes of two trees tends to approach the search tree of the other, which can theoretically enhance the purpose of the search and shorten the time of path planning. The two search trees are set as T_a and T_b , respectively. The search tree T_a is initialized with the current position X_{init} of

USV, and the search tree T_b is initialized with the track recovery point X_{goal} on the original route. In the iteration process, a random point $P_r(X_r, Y_r)$ is generated from the search tree T , and the node closest to the random point P_r in T is $P_p(X_p, Y_p)$. As the parent node in the iteration process, P_p will be extended to P_r with the exploring step S of USV, and the resulting child node is $P_n(X_n, Y_n)$. The obstacle coordinate is $P_o(X_o, Y_o)$, and the expansion radius of the obstacle is R . The execution process of Bi-RRT is shown in Fig. 2.

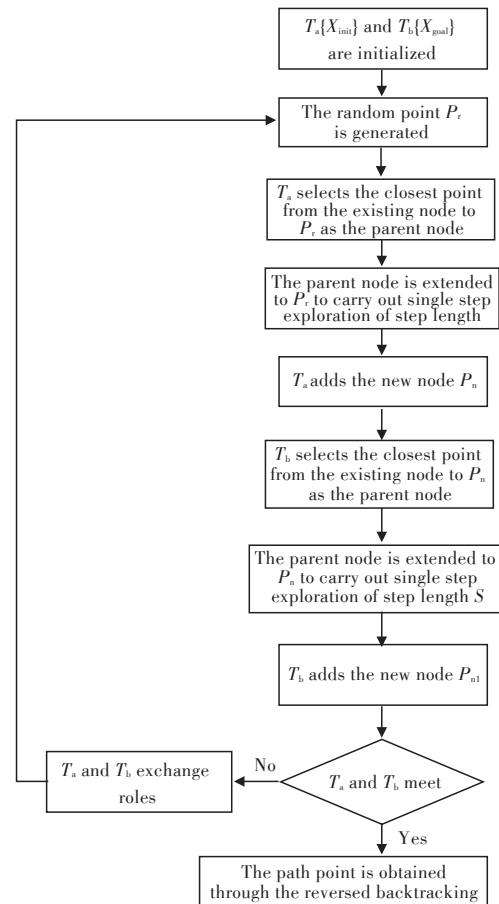


Fig.2 The execution procedure of Bi-RRT algorithm

When the Bi-RRT algorithm is applied to the USV collision avoidance, the two search trees respectively take the current position $X_{init}(X_0, Y_0)$ of vessel A and the track recovery point $X_{goal}(X_1, Y_1)$ on the scheduled route OS as the root node, and then the node is continuously expanded until the two trees satisfy the meeting conditions. Finally, the collision avoidance path point of vessel A is obtained and the path planning is completed by backtracking the encounter points of two search trees respectively. The construction process of Bi-RRT algorithm is shown in Fig. 3.

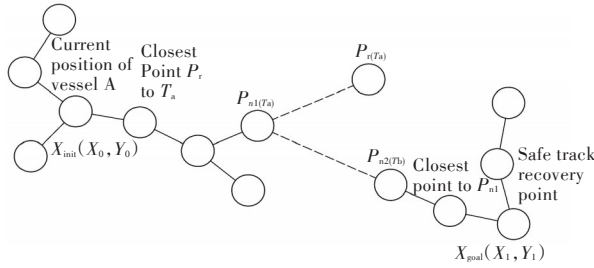


Fig.3 The build process of Bi-RRT algorithm

3 Improvement of Bi-RRT algorithm

Based on the velocity obstacle method, taking into account the actual situation of USV collision avoidance as well as the operational efficiency and real-time performance of the algorithm itself, we improve the Bi-RRT algorithm aiming at the disadvantages that the extension mode of the search tree is not conducive to the intelligent avoidance of obstacle and the selection mode of the parent node affects the real-time performance of the algorithm.

3.1 Design of the obstacle repulsion vector $\mathbf{R}$

The extension method of the search tree of Bi-RRT algorithm is to extend the node which is closest to the random point in the position space in a search tree to the new node firstly, and then to extend the node closest to the new node in another search tree. Although the extension method theoretically strengthens the purpose of the extension of the two trees, if the random point guides the two search trees to approach the obstacle simultaneously, as shown in the USV collision avoidance model, the random point guides the USV course to adjust to the cone collision area. This causes an increase in the probability of falling to the obstacle area after increasing the standard step length of subsequent node extension, an increase in the unnecessary consumption of the algorithm, and the failure in algorithm convergence and planning of an effective collision avoidance path.

As a result, this paper proposes the obstacle repulsion vector $\mathbf{R}$ based on the velocity obstacle method to improve the extension step of Bi-RRT algorithm. After the improvement, it can automatically identify whether the extension direction of the parent node is located in the cone collision area in the iteration process and dynamically adjust the action intensity of $\mathbf{R}$ according to the collision avoidance risk coefficient ω , so as to make the collision avoidance path of

USV more reasonable and efficient.

According to the extension step of the Bi-RRT algorithm, the coordinate $P_n(X_n, Y_n)$ of the extended child node is

$$\begin{cases} X_n = X_p + S \times \cos \left[\arctan \left(\frac{Y_r - Y_p}{X_r - X_p} \right) \right] \\ Y_n = Y_p + S \times \sin \left[\arctan \left(\frac{Y_r - Y_p}{X_r - X_p} \right) \right] \end{cases} \quad (1)$$

The vector expression is shown in Eq. (1)

$$\mathbf{P}_p \mathbf{P}_n = S \times \frac{\mathbf{P}_p \mathbf{P}_r}{|\mathbf{P}_p \mathbf{P}_r|} \quad (2)$$

If the extension operation leads the USV course to the cone collision area, namely geometrically $\alpha_1 < \theta_1$, where

$$\alpha_1 = \arccos \left(\frac{\mathbf{P}_p \mathbf{P}_n \cdot \mathbf{P}_p \mathbf{P}_o}{|\mathbf{P}_p \mathbf{P}_n| \times |\mathbf{P}_p \mathbf{P}_o|} \right) \quad (3)$$

$$\theta_1 = \arcsin \left(\frac{R}{|\mathbf{P}_p \mathbf{P}_o|} \right) \quad (4)$$

At this point, it will trigger the improvement of $\mathbf{R}$ and ω in the Bi-RRT algorithm in this paper. Under the action of $\mathbf{R}$ and ω , the extended child node P_n and its coordinate will no longer meet Eq. (1) and Eq. (2), and P'_n is obtained through the transition vector $\mathbf{V}$ and the vector expression Eq. (8):

$$\mathbf{R} = \omega \times S \times \frac{\mathbf{P}_o \mathbf{P}_p}{|\mathbf{P}_o \mathbf{P}_p|} \quad (5)$$

$$\omega = \left[\tanh \left(\frac{2 \times |\mathbf{P}_p \mathbf{P}_o|}{L_s} \right) \right]^{-1} \quad (6)$$

$$\begin{cases} \mathbf{V} = S \times \frac{\mathbf{P}_p \mathbf{P}_r}{|\mathbf{P}_p \mathbf{P}_r|} + \mathbf{R} \\ \mathbf{P}_p \mathbf{P}'_n = S \times \frac{\mathbf{V}}{|\mathbf{V}|} \end{cases} \quad (7)$$

where L_s is the critical distance of safety collision avoidance. The core function of Eq. (6) is the hyperbolic tangent function $y = \tanh(x)$, the graph of which is shown in Fig. 4.

Eq. (7) shows that when the extension direction of the parent node P_p is within the cone collision area, the USV course is dangerous. At this point, on the basis of the original extension, P_p is added to $\mathbf{R}$ in the connection direction of P_p according to the dynamic regulation of ω , so as to make the search tree extend towards the trend away from the obstacle. Fig. 5 shows the action diagram of $\mathbf{R}$ and ω .

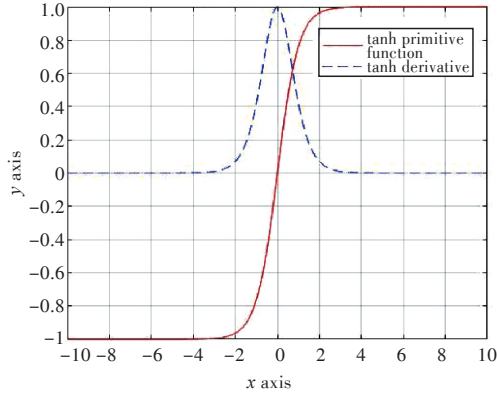
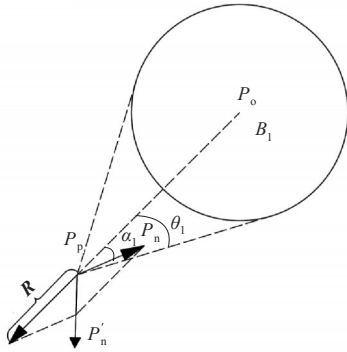


Fig.4 Graph of hyperbolic tangent function


 Fig.5 Schematic diagram of the action between R and ω

As can be seen from Fig. 4, the function $y = \tanh(x)$ located on the positive part of x axis has the following characteristics. 1) When $x \in (0, 2)$ is input, $y \in [0, 1]$ and the value changes dramatically. 2) When $x \in (2, +\infty)$ is input, y is not sensitive to the change of x . When $|P_p P_o| > L_s$, with the increase in $|P_p P_o|$, according to the characteristic 2), the value of ω stays around 1. As shown in the obstacle avoidance of USV, when the distance between the parent node P_p and the center of the obstacle meets a certain safety limit, the effect of R is not obvious. When $|P_p P_o| < L_s$, with the decrease in $|P_p P_o|$, according to the characteristic 1), the value of ω will increase significantly, which is reflected in the obstacle avoidance of USV. When the parent node P_p is closer to the center of the obstacle, the effect of R will be more obvious, making the USV course extend towards the trend away from the obstacle.

3.2 Design of parallel extension and target point attraction vector A

The Bi-RRT algorithm determines the sequence of location and extension of the parent node in the two search trees. Moreover, T_b can only implement the extension step after the new node P_n is generated by T_a . Although the two trees tend to extend

close to each other, the execution of this sequential structure unnecessarily consumes the running time.

Therefore, this paper proposes an improved method that two search trees extend in parallel and have the tendency to approach each other under certain conditions. This improved method generates a random point through two search trees respectively. The extension of the search tree is completed by selecting the extension mode of the parent node respectively according to the random point. In addition, R stimulates the target point attraction vector A under certain conditions, which makes the search tree extend in parallel without losing the advantage of the Bi-RRT algorithm that the two search trees approach each other.

In the iteration process, after P_n is obtained from a search tree based on Eq. (1), if $\alpha_1 > \theta_1$, A in the Bi-RRT algorithm will be improved in this paper. Under the action of A , the extended child node P_n and its coordinate will no longer meet Eq. (1) and Eq. (2), and P_n'' is obtained through the transition vector U and Eq. (19):

$$A = S \times \frac{P_p P_g}{|P_p P_g|} \quad (8)$$

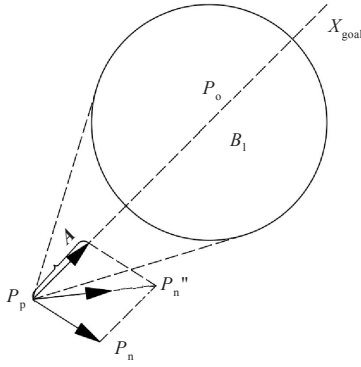
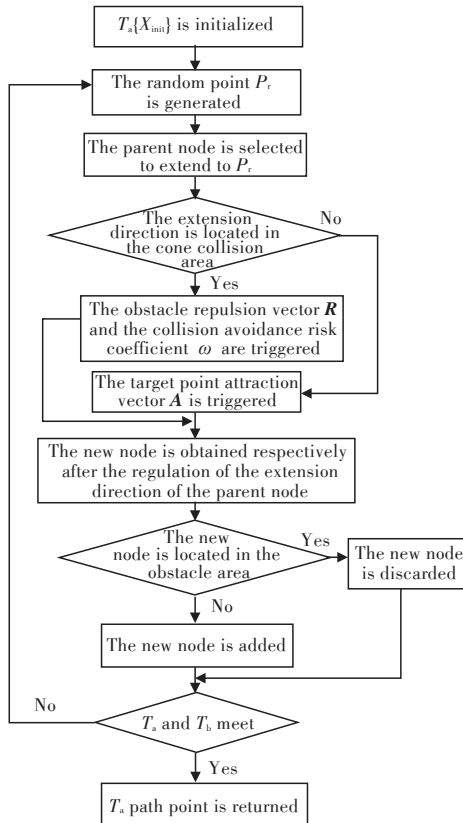
$$\begin{cases} U = S \times \frac{P_p P_r}{|P_p P_r|} + A \\ P_p P_n'' = S \times \frac{U}{|U|} \end{cases} \quad (9)$$

where P_g is the target point coordinate. For the search tree T_a , P_g is the track recovery point X_{goal} , on the original route. For the search tree T_b , P_g is the current position X_{init} of USV.

Eq. (9) shows that when the extension direction of the parent node is outside the cone collision area, the USV course is safe. At this point, P_p is added to A in the connection direction of P_g on the basis of the original extension, so that the search tree will grow towards the target point. Fig. 6 shows the action diagram of the action of A .

3.3 Improvement of Bi-RRT automatic collision avoidance algorithm process

As the two search trees in this paper extend in parallel and do not interfere with each other, with T_a as an example, the process of achieving automatic collision avoidance using the algorithm in this paper is presented, as shown in Fig. 7. Except in the first initialization step where T_b is initialized with the track

Fig.6 Schematic diagram of the action of A Fig.7 The extension procedure of search tree T_a in improved Bi-RRT algorithm

recovery point X_{goal} on the original route, the other processes of T_b are exactly the same as those of T_a .

4 Simulation result and analysis

In order to verify the effectiveness of the improved Bi-RRT algorithm, Microsoft Visual Studio 2017 development environment is used to write C++ program for simulation experiment, and the Bi-RRT algorithm is compared with the improved Bi-RRT algorithm in this paper. In this paper, the high-speed USV described in Reference [14] is selected for simulation experiment. Its navigation speed is 20 kn, and the step length of single step exploration is set as $S=10$.

In the simulation experiment, the scheduled route OS of USV is set as the line segment from point $(0, 0)$ to point $(100, 100)$ on the first quadrant angle bisector of the two-dimensional coordinate system. The current position X_{init} of USV is set as $(40, 40)$ and the track recovery point X_{goal} is $(65, 65)$. The radius of safety area is set as 10 for the obstacle B_1 on OS , and that of obstacles B_2 and B_3 around OS is set as 10 and 15, respectively. In order to test the universality of the algorithm for various collision avoidance environments, we use random number function of C++ program to generate the circle center coordinates O_1 , O_2 and O_3 of B_1 , B_2 and B_3 and then these coordinates are regarded as the algorithm input, and O_1 is restricted on OS and located between X_{init} and X_{goal} ; O_2 and O_3 are no more than 25 away from OS .

Fig. 8 shows the comparison path point planning for automatic collision between basic and improved Bi-RRT algorithm. As can be seen from the figure, the turning point of the path from the collision avoidance path point planned by USV according to the improved Bi-RRT algorithm to the track recovery point X_{goal} is significantly reduced and the path is smoother. This is because the target point attraction vector A is triggered when the extension direction of the parent node is outside the cone collision area, which makes the extension of the two search trees more purposeful and reduces the path oscillation problem caused by the randomness of the algorithm.

As both algorithms are random algorithms, in order to reduce the randomness of the results and improve the credibility of argument, with the collision

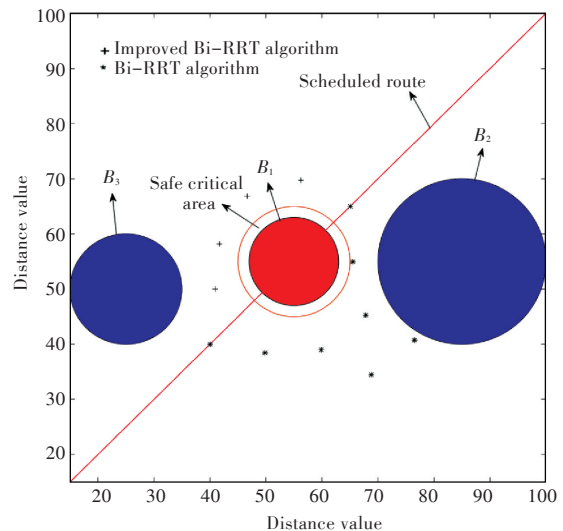


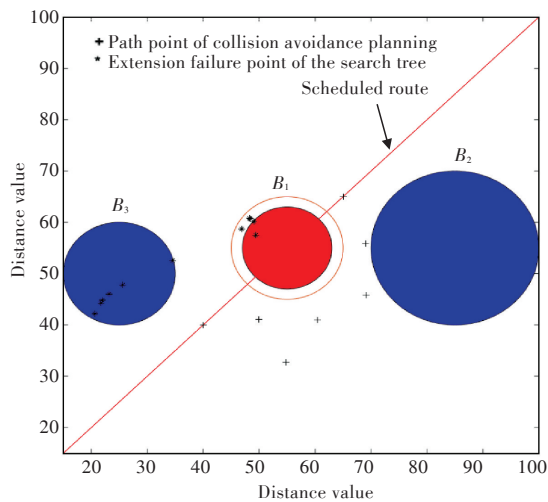
Fig.8 The comparison of path point planning for automatic collision between basic and improved Bi-RRT algorithm

avoidance environment as the input, we run the two algorithms several times so as to compare the number of path point, the times of parent node extension failure, and the time consumed by the algorithm to

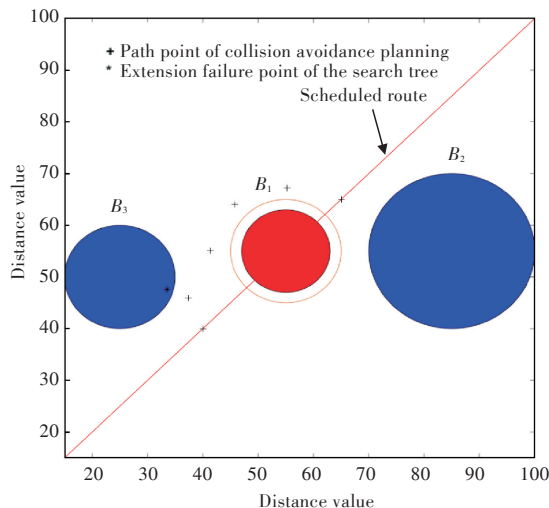
complete the planning of collision avoidance path point. The results are shown in Table 1. Fig. 9 shows the comparison of collision times between basic and improved Bi-RRT algorithms.

Table 1 The comparison of running result between basic and improved Bi-RRT algorithm

No.	Bi-RRT algorithm				Improved Bi-RRT algorithm			
	Number of path point	Times of extension failure of parent node T_a	Times of extension failure of parent node T_b	Time consumption of algorithm /ms	Number of path point	Times of extension failure of parent node T_a	Times of extension failure of parent node T_b	Time consumption of algorithm /ms
1	7	2	4	24	6	0	0	19
2	9	3	5	31	7	0	0	19
3	8	3	4	25	6	0	1	19
4	10	1	4	27	6	0	0	19
5	7	6	11	39	7	1	1	19
6	8	6	8	33	6	2	0	19
7	8	5	7	29	6	0	1	18
8	9	7	4	28	6	1	1	20
9	10	2	5	26	6	0	1	18
10	7	3	9	30	7	0	2	16



(a) Bi-RRT algorithm



(b) Improved Bi-RRT algorithm

Fig.9 The comparison of collision times between basic and improved Bi-RRT algorithm

As the exploration step length of both algorithms is the same, the number of path point can directly reflect the path length. As can be seen from Table 1 and Fig. 8, the path length from the collision avoidance path point planned by USV according to the improved Bi-RRT algorithm to the track recovery point X_{goal} is significantly shorter than the results of the Bi-RRT algorithm under the same conditions. When the navigation speed is constant, the path planned by the improved Bi-RRT algorithm can enable the USV to complete the collision avoidance action in less time and consume less fuel, which is of great significance to the improvement of the endurance of USV during the task execution.

As shown in Table 1 and Fig. 9, the times of extension failure of the parent node in the two search trees T_a and T_b , the one with the improved Bi-RRT algorithm is significantly less than that in the Bi-RRT algorithm, and the number of extension failure node of the search tree in the improved Bi-RRT algorithm is significantly less than that in the basic algorithm, which indicates that the sub-extension node has a greater probability of passing the "collision detection" in the iteration process of the improved Bi-RRT algorithm. This is because the obstacle repulsion vector $\mathbf{R}$ and the collision risk coefficient ω dynamically adjust the extension direction of the parent node, so that the new child node obtained by extension can have a greater probability to be retained, which accelerates the convergence of the algorithm to some extent. According to the comparison in Ta-

ble 1, it can also be found that the improved Bi-RRT algorithm proposed in this paper has significantly less time consumption than the basic algorithm when completing the same planning task of the collision avoidance path. This is because the two search trees extend in parallel and the target attraction vector $\mathbf{A}$ is triggered when the extension direction of the parent node is outside the cone collision area, which improves the real-time performance of the algorithm.

5 Conclusions

In this paper, the Bi-RRT algorithm is combined with the velocity obstacle method. According to the situation that the extension direction of the parent node is located in the cone collision area, the obstacle repulsion vector $\mathbf{R}$ and the collision avoidance risk coefficient ω are proposed to dynamically adjust the extension direction of the parent node, so as to make the search tree extend to the trend of moving away from the obstacle and make the new child node obtained from the extension have a greater probability to be retained, thus accelerating the convergence of the algorithm. In addition, the method of parallel extension of two search trees is proposed. The target attraction vector $\mathbf{A}$ is triggered when the extension direction of the parent node is outside the cone collision area, which makes the extension of the two search trees more purposeful and reduces the path oscillation problem caused by the randomness of the algorithm. The improved algorithm can be applied to the dynamic obstacle avoidance problem of USV with higher real-time requirements in the future, as well as the track rolling planning problem of USV returning to the originally scheduled route under the continuous disturbance of large wind wave.

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[Continued on page 42]

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关键词: 无人水面艇; 自主避障; 双向搜索树算法; 速度障碍法