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Modeling and simulation of five DOF motions for SWATH ships

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Abstract: To develop marine simulators of Small Waterplane Area Twin Hull (SWATH) ships and gain a better understanding of the maneuverability of the SWATH, a mathematical model with five Degree of Freedom (DOF) for SWATH has been established, in which the surge, sway, yaw, roll and pitch motions are all considered. The characteristics of the SWATH ship form and the hydrodynamic interaction among the twin bodies, twin propellers and twin rudders are taken into consideration on the basis of the three DOF maneuvering mathematical model. To verify the effectiveness of the modeling and ensure the use of the model in marine handling simulators, a detailed model is tentatively established on the basis of empirical formulas. Based on the acquired results, tests were performed to simulate the steady turning and zigzag motions of the ship. The results are in accordance with regular pattern and trend of motion of SWATH ships. This verifies the effectiveness of the mathematical model of the turning movement.

Key words: Small Waterplane Area Twin Hull (SWATH); five degree of freedom; marine simulator; numerical simulation

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0 Introduction

Small Waterplane Area Twin Hull (SWATH) ship is the first choice for navigation and operation under high sea conditions, whose military and civil application prospects have been paid more and more attention by the relevant departments of China and abroad. Compared with the conventional ships, SWATH ship features advantages of high speed, excellent seakeeping and good stability, but it also has disadvantages such as larger frictional drag and large turning radius. These abnormal maneuverability characteristics of SWATH have put forward higher requirements for the maneuvering skills of the operators, brought new problems to the education and training of the operators, and meanwhile brought challenges to the traditional maritime transport safety. At present, the recognized method of solving the above problems is to use the marine simulator to carry out study and training for the maneuverability characteristics and skills of new ship. The core problem of SWATH simulator development is to establish a comprehensive and accurate mathematical model of SWATH ship motion.

The majority research on SWATH of China and abroad is concerned with the seakeeping and longitudinal motion control system[1-2]. As of now, mathematical modeling research on the SWATH motion is still less, which mainly focuses on the three degree of freedom (DOF) motion of SWATH. For example, Zhang et al.[3] established a three DOF motion equation for SWATH and it was applied in marine simulator; Wang et al.[4] and Xiong et al.[5] established the motion model for SWATH and gave the forecast on its maneuverability; Zhang et al.[6] completed the four DOF maneuvering motion simulation for SWATH. Therefore, on the basis of previous studies and the development of SWATH marine simulator, in order to fully explain the law of motion of the SWATH ship and increase the fidelity in the simulator, the model

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of five DOF motion for SWATH coupling of surge, sway, yaw, roll and pitch motions was established in this paper using separation method of Ship Maneuvering Mathematical Model Group (MMG), so as to reflect the motion response characteristics of SWATH.

1 Five DOF motion model for SWATH

It is assumed that SWATH navigates in the water area of infinite depth and width; the hull is rigid body; no waves and current exist; only motion in the constant velocity domain is studied in the modeling process; meanwhile, the coupling effect of ship's slight motion is not considered, such as the impact of rolling on horizontal and vertical hydrodynamic coefficients.

As shown in Fig. 1, this study adopted the inertial coordinate system $O_o-X_oY_oZ_o$ that describes parameters of ship location and heading angle, and the horizontal body coordinate system *o*–*xyz* which solves the equation of motion control and describes the ship's velocity angle and velocity. The origin was set at the center of gravity. It is defined that *ox* axis pointed to the bow; *oy* axis pointed to the starboard; *oz* axis pointed to the keel; and the positive direction was the right rudder. The barycenter coordinate of the ship in the inertial coordinate system was denoted as (x_o, y_o, z_o).

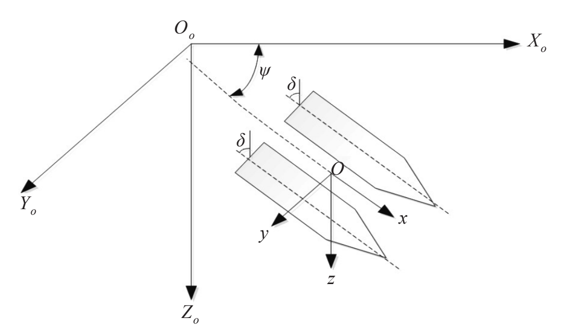


Fig.1 Coordinate systems of SWATH ship maneuvrability

In the above coordinate systems, according to the MMG separation method[7], on the basis of the three DOF equations, the ship's roll and pitch motions were added, and the five DOF motion formula of the SWATH ship was established.

$$\begin{cases}(m+m_x)\dot{u}-(m+m_y)vr=X_{\rm H}+X_{\rm R}+X_{\rm P}\\(m+m_y)\dot{v}+(m+m_x)ur=Y_{\rm H}+Y_{\rm R}+Y_{\rm P}\\(I_{xx}+J_{xx})\dot{p}\approx L_{\rm H}+L_{\rm R}+L_{\rm P}\\(I_{yy}+J_{yy})\dot{q}\approx M_{\rm H}+M_{\rm R}+M_{\rm P}\\(I_{zz}+J_{zz})\dot{r}=N_{\rm H}+N_{\rm R}+N_{\rm P}\end{cases} \tag{1}$$

In the equation, m, m_x, and m_y are the mass of the ship and additional mass of the ship on the *x* and *y* axes; I_{xx}, I_{yy}, and I_{zz} are the rotational inertia of the ship around the *x*, *y*, *z* axes; J_{xx}, J_{yy}, and J_{zz} are respectively the additional moment of inertia of the ship around the *x*, *y*, *z* axes; *u*, *v*, *p*, *q* and *r* are the forward velocity, transverse velocity, rolling velocity, pitching velocity and yawing angular velocity in each direction of the ship, and the points on them are the derivatives of time, i.e. their motion acceleration; *X*, *Y*, *L*, *M* and *N* are the hydrodynamic force and moment of surge, sway, roll, pitch and yaw motions on the ship, and the subscripts H, R, and P are forces of the hull, rudder and propeller.

2 Calculation of the force and moment of SWATH

2.1 Additional mass and additional moment of inertia

The additional mass and additional moment of inertia of SWATH were calculated. Firstly, the win bodies of SWATH were separated, and the additional mass and moment of inertia of the SWATH's twin bodies were solved. Then, the additional mass and moment of inertia of the SWATH ship were fitted.

The additional mass and moment of inertia of a single body can be calculated according to the formula and map of the additional mass and moment of inertia of an ellipsoid with a long axis of 2*a* and a short axis of 2*b* in Ref. [8].

In this paper, the half–lengths of the two axes of the ellipsoid are as follows:

$$a=\frac{L}{2}, b=d \tag{2}$$

where *L* is the ship length, *d* is ship draft. Thus, the additional mass and moment of inertia of SWATH can be obtained as follows:

$$\begin{cases}m_x=2m_{lx}(1+\frac{3}{16}(\frac{0.5B}{C})^3)\\m_y=m_{ly}(1+\frac{3}{8}(\frac{d}{C})^3)\\J_{xx}=2(m_{lx}(1+\frac{3}{16}(\frac{0.5B}{C})^3C^2)+J_{lxx}(1+\frac{3}{8}(\frac{d}{C})^3))\\J_{yy}=2(m_{lx}(1+\frac{3}{16}(\frac{0.5B}{C})^3C^2)+J_{lyy}(1+\frac{3}{8}(\frac{d}{C})^3))\\J_{zz}=2(m_{lx}(1+\frac{3}{16}(\frac{0.5B}{C})^3C^2)+J_{lzz}(1+\frac{3}{8}(\frac{d}{C})^3))\end{cases} \tag{3}$$

In the equation, *C* is the distance between the twin bodies, *B* is the body width, m_{lx}, m_{ly}, J_{lxx}, J_{lyy} and J_{lzz} are the additional mass and moment of inertia of a single body.

2.2 Hull force

In Eq. (1), the longitudinal force X_{H} is based on the Inoue model[9], which can be expressed as

$$X_{\mathrm{H}}=X(u)+X_{vv}v^2+X_{vr}vr+X_{rr}r^2 \tag{4}$$

In the equation, $X(u)$ is ship resistance, X_{vv}, X_{vr} and X_{rr} are the second-order hydrodynamic derivatives of the longitudinal force.

According to the characteristics of SWATH ship, the lateral force Y_{H} and yawing moment N_{H} are calculated according to the calculation method of slender body theory proposed in Ref. [5], and the force of a single body is calculated as

$$\begin{cases} Y_{\mathrm{H}}=-\dfrac{\rho}{2}uvLd\dfrac{\pi}{2}\lambda A^*\eta+\dfrac{\rho}{2}urL^2d\dfrac{\pi}{4}\lambda A^*\eta \\ N_{\mathrm{H}}=-\dfrac{\rho}{2}uvL^2d\dfrac{\pi}{2}\lambda A^*(2-\eta)-\dfrac{\rho}{2}urL^3d\dfrac{\pi}{8}A^*\eta \end{cases} \tag{5}$$

In Eq. (5), λ, A^* and η are calculated by the following formulas:

$$\begin{cases} \displaystyle\int_L \frac{A(x)}{x}\mathrm{d}x=-A^*\eta \\ \lambda=\dfrac{2d}{L} \end{cases} \tag{6}$$

In Eq. (5) and Eq. (6), ρ is mass density of fluid, λ is the symbol for simplification, representing $2d/L$ as λ, $A(x)$ is dimensionless additional inertial coefficient of the cross section at x, which is the ratio of additional mass of the section to that of the disc with draft d as the radius. Here, $A(x)$ is considered to be uniformly distributed, and its value is A^*, η is the empirical correction coefficient.

The heeling moment L_{H} and the pitching moment M_{H} of SWATH are calculated as follows

$$\begin{cases} L_{\mathrm{H}}\approx L_{pp}|p|p-mgGM\sin\varphi \\ M_{\mathrm{H}}\approx M_{qq}|q|q-mgGM_{\mathrm{L}}\sin\theta \end{cases} \tag{7}$$

In the equation, L_{pp} and M_{qq} are the nonlinear hydrodynamic coefficient, GM and GM_{L} denotes the horizontal and vertical metacentric heights of the ship, g is the acceleration of gravity; φ is the roll angle, (°), and θ is the pitch angle, (°).

2.3 Propeller force

For the convenience of calculation, the calculation of surge, sway, roll, pitch and yaw forces of the SWATH propeller can use the calculation method of single propeller force, but the effect of the twin propellers needs to be considered, which can be calculated according to the following formulas:

$$\begin{cases} X_{\mathrm{P}}=(1-t_{\mathrm{p}})(T_{(\mathrm{p})}+T_{(\mathrm{s})}) \\ N_{\mathrm{P}}\approx(1-t_{\mathrm{p}})(T_{(\mathrm{p})}-T_{(\mathrm{s})})\cdot\dfrac{1}{2}C \\ Y_{\mathrm{P}}\approx L_{\mathrm{P}}\approx 0 \\ M_{\mathrm{P}}\approx X_{\mathrm{P}}z_{\mathrm{P}} \\ T_{(\mathrm{p})}=\rho n|n|D_{\mathrm{p}}{}^4K_{\mathrm{T}}(J_{\mathrm{p}}) \end{cases} \tag{8}$$

In the equation, $T_{(\mathrm{p})}$ and $T_{(\mathrm{s})}$ are the thrust of the left and the right propellers, and their calculation formulas are the same, t_{p} is thrust deduction coefficient, n is the propeller's rotational velocity, D_{P} is propeller diameter, K_{T} is thrust coefficient, which is a function of advance coefficient J_{P} of the propeller, and the calculation method can refer to the formulas of twin-propeller ship in Ref. [7], z_{P} is hull coordinate value of the propeller on the z axis.

2.4 Rudder force

Considering the distance between the two rudders of SWATH and the interaction between the hull and the propeller, the corrected formula of the rudder force is as follows:

$$\begin{cases} X_{\mathrm{R}}=(1-t_{\mathrm{R}})(F_{\mathrm{N(p)}}+F_{\mathrm{N(S)}})\sin\delta \\ Y_{\mathrm{R}}=(1+a_{\mathrm{H}})(F_{\mathrm{N(p)}}+F_{\mathrm{N(S)}})\cos\delta \\ L_{\mathrm{R}}=-\left(z_{\mathrm{R}}+a_{\mathrm{H}}z_{\mathrm{H}}\right)(F_{\mathrm{N(p)}}+F_{\mathrm{N(S)}})\cos\delta\approx \\ \qquad -(1+a_{\mathrm{H}})(F_{\mathrm{N(p)}}+F_{\mathrm{N(S)}})z_{\mathrm{R}}\cos\delta \\ M_{\mathrm{R}}\approx 0 \\ N_{\mathrm{R}}=-(1-t_{\mathrm{R}})(F_{\mathrm{N(p)}}-F_{\mathrm{N(S)}})\sin\delta\cdot\dfrac{1}{2}C- \\ \qquad (1+a_{\mathrm{H}})x_{\mathrm{R}}(F_{\mathrm{N(p)}}+F_{\mathrm{N(S)}})\cos\delta \end{cases} \tag{9}$$

In the equation, $F_{\mathrm{N(p)}}$, $F_{\mathrm{N(S)}}$ are the normal forces of port and starboard rudders , both of which are calculated using the Fujii formula in this paper[10], t_{R} is resistance deduction coefficient of rudder, a_{H} is the correction factor of rudder force, x_{R} and z_{R} are the coordinates of rudder force center; z_{H} is the vertical distance from the center of transverse force of steering induced hull to the center of gravity of the ship.

2.5 Calculation model of steering engine

For the motion response of the steering engine, the first-order inertia element was used for approximate processing.

$$T_{\mathrm{R}}\dot{\delta}=\delta_{\mathrm{E}}-\delta \tag{10}$$

In the equation, T_{R} is the response time of the steering engine, which is taken as 2.5 s, δ is the current rudder angle, and the point on it represents the derivative of time, δ_{E} is the target rudder angle.

3 Numerical simulation

To verify the accuracy of the model, a SWATH was selected as an example, and its five DOF equation was established. The fourth-order Runge-Kutta method was used to solve the differential equation, and simulation verification was carried out for the law of motion of turning and zigzag. The parameters of the SWATH are shown in Table 1, and other model parameters were estimated using empirical formulas.

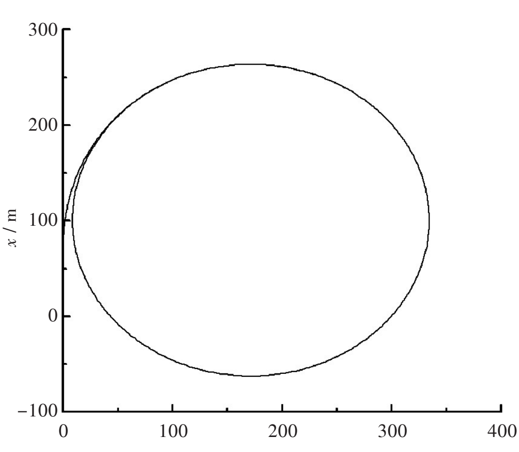
Table 1 Particulars of a SWATH ship

Name	Value
Ship length/m	35.9
Length between perpendiculars/m	31.5
Total width/m	17.1
Demihull spacing/m	12.3
Moulded depth/m	5.85
Draft/m	3.15
Design speed/kn	26.5
Main engine power/ kW	2 978×2
Horizontal metacentric height/m	≈0.06
Vertical metacentric height/m	≈4.84

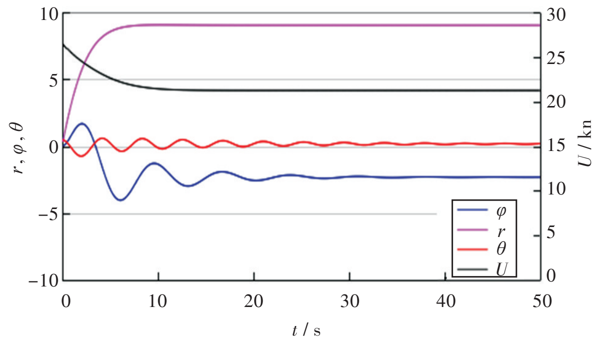
3.1 Turning motion simulation of SWATH

According to the design parameters of SWATH, the initial state of the simulation is as follows: the speed of the main engine was set to 270 r/min, the navigational speed was 26.5 kn; the twin rudders were set to +35° for the right turning motion simulation; and the simulation results are shown in Fig. 2, where r is the angular speed of yawing, (°)/s, and U is navigational speed, kn.

Under the condition of lacking the turning results of the SWATH ship, the qualitative analysis of the simulation results shows that the SWATH turning diameter is larger, about 350 m, which accords with the SWATH turning characteristics of about 10 times the ship length because the course stability is good and the turning diameter is generally large[11]. The roll and pitch phenomenon in the turning is as follows: at the beginning, the ship is in tumblehome state; with the increase of angular velocity of yawing, it became flared; then it tended to be stable; at the same time, the ship started to show regular slight bow lift and arch nose phenomenon.



(a) Right turning trajectory

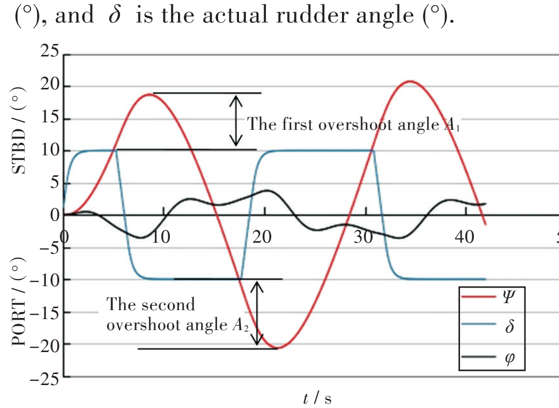


(b) Time history curve

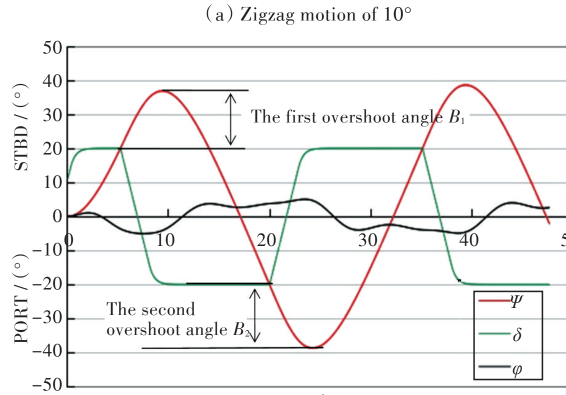
Fig.2 The right turning simulation of a SWATH ship

3.2 Zigzag maneuvering motion simulation

The motion simulation of zigzag maneuvering of the SWATH was carried out. The initial speed was set to 26.5 kn, and zigzag motion test simulation of 10° and 20° was conducted. The simulation results are shown in Fig. 3, where Ψ is the yawing angle (°), and δ is the actual rudder angle (°).



(a) Zigzag motion of 10°



(b) Zigzag motion of 20°

Fig.3 Zigzag maneuvring simulation of a SWATH ship

As Fig. 3 shown that, in the zigzag simulation of 10°/10° of SWATH, the first and the second overshoot angles A_1 and A_2 were 8° and 11.5°; in the zigzag simulation of 20°/20°, the first and second overshoot angles B_1 and B_2 were 16.8° and 18.7°, and the SWATH had large turning inertia; in addition, in the zigzag motion process, the roll angle of the SWATH will have irregular changes.

4 Conclusion

In this paper, based on the studies on the mathematical model of SWATH marine simulator, using the MMG's separation method, according to the three DOF model for SWATH, and combined with the characteristics of SWATH ship form, a mathematical model of five DOF motion was established containing roll and pitch motions, and the corresponding calculation formulas were achieved. Then by selecting an real SWATH ship, the specific motion equation was established and fourth-order Runge-Kutta method was used for solving the differential equations. The simulation of the right turning motion and zigzag motion for SWATH was achieved. Further, the simulation results were qualitatively analyzed, and the results are in good agreement with the maneuvering motion principle of SWATH and the actual law of motion, which proved that the established model can be used in marine simulator. In the future, based on this, the influence of wind, wave and current on SWATH motion will be explored and the six DOF motion equation for SWATH will be established to improve the fidelity of SWATH simulator.

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小水线面双体船五自由度运动建模与仿真

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摘　要：[**目的**] 为了提高航海模拟器中小水线面双体船(SWATH)的模拟精度，更好地掌握此类船舶的操纵特性，[**方法**] 根据SWATH船型的运动特点，在MMG三自由度模型的基础上，较为完整地计入耦合的纵荡、横荡、艏摇、横摇及纵摇运动，建立SWATH的五自由度运动数学模型。基于此应用模型建立SWATH的实船数学模型，对该数学模型进行微分求解，仿真模拟SWATH的旋回运动、Z形运动，并进行定性分析。[**结果**] 分析显示：仿真模拟结果符合SWATH操纵运动原理及实际运动特征，验证了数学模型的准确性。[**结论**] 该模型可真实反映实船的运动规律，可应用于航海模拟器中。

关键词：小水线面双体船；五自由度；航海模拟器；数值仿真