**DOI:** 10.3969/j.issn.1673-3185.2017.01.010 **http:** // english.ship-research.com

**Translated from:** ZHANG Z Y, ZHAO Y. Reliability calculation method of longitudinal ultimate strength of ships under extreme sea conditions[J]. Chinese Journal of Ship Research, 2017, 12(1): 63-71.

# Reliability calculation method of longitudinal ultimate strength of ships under extreme sea conditions

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**Abstract:** Building a large ship in an overall sea area is a trend in shipbuilding. As such, it is necessary to take into account the influence of extreme waves in the calculation of a ship's longitudinal ultimate strength reliability. The general method of load calculation does not take into account the effects of special wave loads under extreme sea conditions. In addition, for reliability analysis, extreme loads have more complicated random variables. The general method of ship reliability calculation requires these variables to obey a certain distribution, which may mean that the original method cannot be used. From the perspective of navigational limit, the maximum value of the wave bending moment is greater than that of the conventional wave bending moment which does not take the impact of extreme wave sea conditions into account. The experimental data show that the wave moment calculation method considering extreme wave sea conditions can to some extent reflect the wave loads of ships more realistically. Secondly, by considering the characteristics of different reliability calculation methods and using case calculations, this paper gives a selection of calculation methods of the longitudinal ultimate strength reliability of ships under extreme sea conditions.

**Key words:** extreme wave conditions; extreme load; longitudinal ultimate strength; reliability calculation

**CLC number:** U661.43

## 0 Introduction

Under the trend of large ships in an overall sea area, the extreme sea conditions encountered in ship navigation need to be paid attention to, which puts forward higher requirements for longitudinal strength of ship. The reliability analysis of longitudinal ultimate strength of ship is the focus of the engineering field, which mainly involves 3 aspects: structural strength, load and reliability analysis. However, compared with the reliability analysis of general ship structure, the selection of random variables of structural strength under extreme conditions, composition of wave load under extreme conditions, as well as the selection and calculation of reliability method considering the ultimate strength and extreme load are worthy of study.

The key for the calculation of structural strength of ship under extreme conditions lies in the extraction of factors that have great influence on the longitudinal ultimate strength of ship, Zhao[1] determined the random variables that have great influence on the longitudinal ultimate strength of ship through the sensitivity analysis of influencing factors to the structure, and by the investigation on the related research in China and abroad, he gave the recommended value of random variable's statistical characteristic value for reliability calculation. The analysis of extreme load under extreme sea condition mainly concerns the influence of this sea condition on wave bending moment, which has significant random characteristics under extreme sea condition. Yu et al.[2] believed that long-term extreme wave bending moment complies with Gumbel I type distribution, but the calcu-

**Received:** 2016 - 03 - 31

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lation method did not involve the extreme sea condition that may be encountered during the voyage in the all-weather large ship trend, and the sea condition has a great influence on the longitudinal wave bending moment of ships. In addition, for the reliability calculation of longitudinal ultimate strength of ship under extreme wave conditions, the 2 basic random variables in limit state function, the wave bending moment and longitudinal ultimate strength of structure under extreme sea conditions do not comply with simple distribution, which makes the limit state function more complicated, resulting in that the common reliability calculation methods may not be applied.

For the above problems, this paper will select 3 types of sea conditions that have great influence on longitudinal ultimate wave bending moment, i.e., slamming, green water and sharp pitching in the extreme sea conditions. From the perspective of navigational boundary of ship, by using the ship navigational boundary rules, and with reference to the wave load calculation methods in the DNV guide for reliability calculation of ship structure[3], a calculation method of extreme wave bending moment was proposed considering the above 3 kinds of sea conditions; and through the investigation of characteristics of different reliability calculation methods, efficient and reasonable reliability calculation method of longitudinal ultimate strength of ship in extreme wave conditions was selected.

# 1 Wave bending moment under extreme wave conditions

The specific steps of the general method for solving the long-term extreme value of wave bending moment are shown in Fig. 1. Firstly, the spectral analysis method was used to predict short-term wave bending moment. Based on the numerical calculation methods such as strip theory or potential flow theory, frequency response function of wave bending moment of ship was obtained; and combined with the wave spectra, for short-term wave sea conditions determined by any significant wave height $H_{s_i}$ and wave zero-crossing period $T_{z_i}$ in wave scatter diagram, the eigenvalues of wave bending moment were solved, such as significant value $M_{s_i}$ of short-term wave bending moment; after the short-term wave bending moment was obtained, with reference to the method in the DNV guide for reliability calculation of ship structure for long-term extreme wave bending moment through the short-term wave bending moment fitting[3], the eigenvalue of long-term wave bending moment distribution for calculating the reliability of longitudinal strength was obtained.

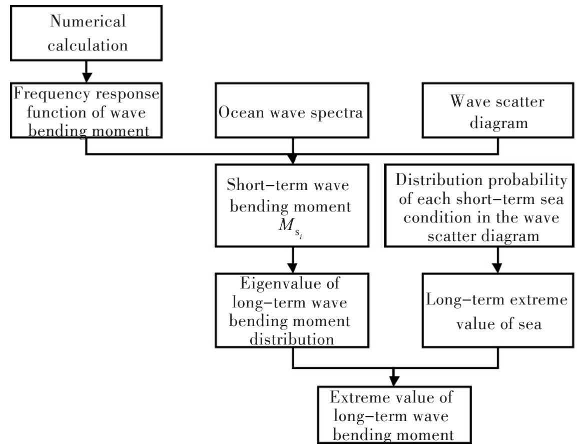


Fig.1 General method to predict wave load

In the trend of large ships in the overall sea area, the study of extreme sea condition affecting the longitudinal ultimate strength should be added to the study of the reliability of longitudinal ultimate strength of ship. This paper selected three types of sea conditions, i.e., slamming, green water and sharp pitching which have great influence on the longitudinal strength of ship, starting from the perspective of navigational boundary, extreme sea condition was introduced into the computation of extreme wave bending moment (Fig. 2).

From Fig. 2 we can see that the calculation meth-

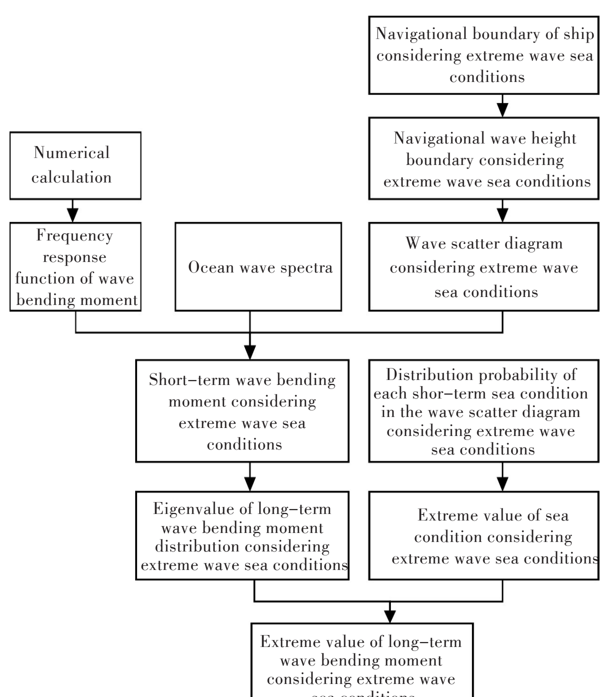


Fig.2 Prediction of extreme wave load under extreme sea conditions

od of extreme wave bending moment under extreme wave sea conditions is different from that of the long-term extreme value of general wave bending moment, and the difference mainly lies in the wave scatter diagram after the extreme sea conditions (green water, slamming and sharp pitching) are taken into consideration, namely, data of sea conditions are different; after considering the navigational boundary of extreme sea condition, the wave height $H_{m_i}$ can be determined according to the navigational boundary, and thereby to delineate data in the wave scatter diagram considering the extreme sea conditions; based on the new data, eigenvalue of the long-term wave bending moment distribution and the long-term extreme value considering 3 types of extreme sea conditions can be fitted again.

To calculate the wave bending moment under extreme wave conditions, we must first find the navigational boundary value under the sea condition, and used these values to determine the navigational wave height boundary, thus to delineate data of sea condition according to the wave height boundary in the wave scatter diagram considering green water, slamming and sharp pitching.

## 1.1 Considering the influence of extreme sea conditions

In order to consider the impact of green water, slamming, sharp pitching and other extreme sea conditions on wave bending moment, starting from the perspective of navigational boundary, we found the wave height boundary determined by the navigational boundary of these special sea conditions, which can be used to select data of sea condition considering the 3 kinds of special sea wave conditions in the wave scatter diagram. Fig. 3 shows the determination of extreme sea condition data. Referring to Fig. 3, firstly, the navigational boundaries of green water, slamming and sharp pitching were selected. According to the relationship between navigational boundary and navigational wave height boundary $H_{m_i}$, the corresponding wave height boundaries $H_{m_1}$, $H_{m_2}$ and $H_{m_3}$ of extreme sea conditions of green water, slamming and sharp pitching were determined. Wave height boundary is the corresponding minimum significant wave height when the green water probability, slamming probability and sharp pitching angle reach the navigational boundaries, that is, the critical probabilities of green water, slamming and sharp pitching.

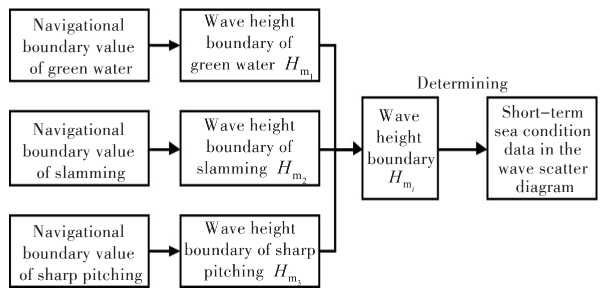


Fig.3 Determining of the extreme wave sea conditions data

By comparing the above wave height boundaries, $H_{max}$ was selected as the wave height boundary of the extreme wave sea conditions, and the data of extreme sea conditions in the wave scatter diagram were determined according to the wave height boundary considering green water, slamming and sharp pitching.

Navigational boundary is the limit seakeeping criterion in the navigation of ships. For the selection of navigation boundaries, the reference values of boundary probability of green water, boundary probability slamming and boundary angle of sharp pitching are given in Table 1.

**Table 1 Navigation criteria of the extreme sea conditions like slamming, green water and sharp pitching**

| No. | Bondary value | | | | | | Source of reference value |
|---|---|---|---|---|---|---|---|
| | Slamming | | Green water | | Sharp pitching | | |
| 1 | Number of slamming in 100 oscillations | 3 | Times of green water per minute | 0.5 | Single significant pitching $\phi_{a1/3}$ /(°) | 4.8 | Ref[4] |
| 2 | Critical probability of slamming | 0.01~0.03(merchant ship)<br>0.03(naval ship)<br>0.03(small boat) | Critical probability of deck wetness | 0.05(merchant ship)<br>0.05(naval ship)<br>0.05(small boat) | – | | Ref[5] |
| 3 | Probability of slamming | 0.01~0.03 | Probability of deck wetness | 0.05 | Pitching/(°) | 3.5 | Ref[6] |
| 4 | Critical probability of slamming | 0.02 | Critical probability of green water | 0.04 | Pitching/(°) | 3.5 | Ref[7] |

Based on the practice of Fig. 3, the relationship between the 3 navigation boundary values and the corresponding wave height boundary $H_{m_i}$ was established after the appropriate navigation boundary values were selected.

Although the original long-term distribution of wave load depends on the calculation in each course, basically the extreme value of load is only related to the heading wave conditions[3], and the navigational speed under the extreme sea conditions is difficult to maintain, therefore, in the calculation, we assume that the ship has zero speed and is in heading wave condition.

1) Green water of a ship deck is defined as that the relative displacement from calculated point of bow to wave surface exceeds the freeboard. The probability of green water is set to $q_1$, which is calculated as[8]:

$$q_1 = \exp\left(\frac{-f^2}{2\bar{R}_1^2}\right) \quad (1)$$

where the freeboard height of bow is $f$ and $\bar{R}_1$ is the standard deviation of the vertical relative motion at the bow. Because the heading wave was selected as the state for computation, the front endpoint of the intersection between the bow and the designed waterline is selected as the calculated point. The motion frequency response function was calculated by using the numerical method based on slice theory or potential flow theory, and then the standard deviation of the motion response can be solved by spectral analysis. The response function of the relative motion between the bow and the wave surface is formed by the superposition of motion response functions of single degree of freedom (DOF), and these response functions of single DOF can be obtained by numerical method. The theoretical formulas of superstition are as follows:

$$H_{\mathrm{RM}(z)} = H_{\mathrm{AM}(z)} - H_{\mathrm{wave}(z)} \quad (2)$$

$$H_{\mathrm{AM}(z)} = H_{\mathrm{heave}} - xH_{\mathrm{pitch}} + yH_{\mathrm{roll}} \quad (3)$$

In the formulas: $H_{\mathrm{RM}(z)}$ is the relative vertical motion displacement; $H_{\mathrm{AM}(z)}$ is the vertical motion displacement; $H_{\mathrm{wave}(z)}$ is the wave height; $H_{\mathrm{heave}}$ is the heave value; $H_{\mathrm{pitch}}$ is the pitch value; $H_{\mathrm{roll}}$ is the rolling value; $x$ and $y$ are the coordinates of calculated point.

According to Formula (1), when the boundary probability of green water is $p_1$, the wave height boundary $H_{m_1}$ of green water is

$$H_{m_1} = \frac{f}{\bar{R}_1\sqrt{2\times\ln\left(1/p_1\right)}} \quad (4)$$

2) The bottom slamming is defined as the situation that the relative velocity of the hull bottom emerging the water surface and again slamming the surface of the water is more than the critical velocity. The probability of the occurrence of the slamming of the bottom is set to $q_2$, and there is the following relation [8]

$$q_2 = \exp\left(\frac{-d_{\mathrm{f}}^{\,2}}{2\bar{R}_1^{\,2}} + \frac{-v_{\mathrm{cr}}^{\,2}}{2\bar{R}_2^{\,2}}\right) \quad (5)$$

where $d_{\mathrm{f}}$ is the draft of bow; $v_{\mathrm{cr}}$ is the critical value of relative velocity when bow bottom emerges the water surface and slams the water surface again, generally $v_{\mathrm{cr}} = 0.09\sqrt{gL}$; $L$ is the length between perpendiculars; $\bar{R}_2$ is the standard deviation of relative velocity of bow under the unit wave height. Because the state of heading wave is used for calculation, similarly, the front endpoint of intersection of bow and designed waterline was selected as the calculated point, the motion frequency response function was obtained using numerical method, and standard deviation of velocity was achieved by spectral analysis. The response function of velocity can be obtained by the vertical motion response function

$$H_V = \mathrm{i}\omega H_{\mathrm{RM}(z)} \quad (6)$$

In the formula: $H_V$ is the velocity response of the calculated point; $\omega$ is the wave encounter frequency.

According to Formula (5), when the boundary probability of hull bottom slamming was $p_2$, there is the wave height boundary $H_{m_2}$ of slamming:

$$H_{m_2} = \frac{1}{\sqrt{2\times\ln\left(1/4p_2\right)}}\sqrt{\left(\frac{d_{\mathrm{f}}}{\bar{R}_1}\right)^2 + \left\{\frac{\left(v_{\mathrm{cr}}/\sqrt{gL}\right)\times\sqrt{gL}}{\bar{R}_2}\right\}^2} \quad (7)$$

3) The sharp pitching of ships is defined as the situation that a part of the propeller blade emerges the water surface. The wave height boundary of sharp pitching is[8] $H_{m_3}$.

$$H_{m_3} = \frac{\phi}{2\bar{R}_3} \quad (8)$$

where $\phi$ is boundary angle of sharp pitching; $\bar{R}_3$ is the standard deviation of hull pitching with unit wave height. The standard deviation of pitching can be obtained using spectral analysis method after solving the response function of pitching by numerical method.

4) Data of the extreme wave sea condition considering green water, slamming and sharp pitching.

According to the above Formulas (4), (7) and (8), the wave height boundary $H_{m_1}$ of green water, the wave height boundary $H_{m_2}$ of slamming and the wave height boundary $H_{m_3}$ of sharp pitching were obtained. The maximal value $H_{max}$ was selected as the calculated wave height boundary. The navigational boundary was the limit index value for a ship to work. Assuming $H_s$ was significant wave height, it can be seen that, green water, slamming and pitching caused by the sea condition of $H_s \geqslant H_{max}$, which was determined by navigational wave height boundary, had great influence on the longitudinal strength of the navigating ship. Therefore, the part in the North Atlantic wave scatter diagram[9] of significant wave height $H_s \geqslant H_{max}$ was selected as the wave scatter diagram considering the extreme sea conditions to calculate the extreme wave loads. The data of extreme wave sea condition selected in the wave scatter diagram are shown in Fig. 4.

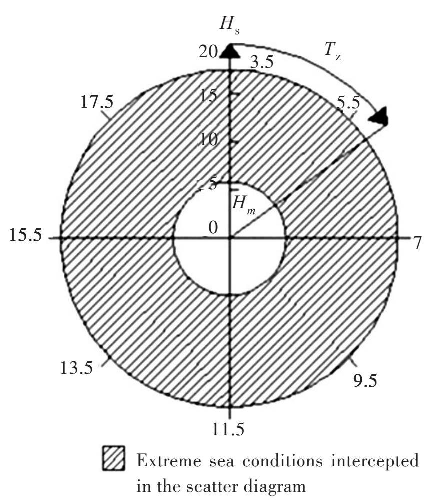


Fig.4 Extreme sea conditions that consider the impact of green water, slamming and sharp pitching

In Fig. 4, the circumferential direction is zero-crossing period $T_z$ of wave. Any short-term sea condition determined by the significant wave height and wave zero-crossing period corresponds to one point in the figure. The overall area in the figure represents the whole short-term sea conditions in the wave scatter diagram. The shadow part, which was determined by the wave height boundary $H_m$ considering extreme sea conditions of green water, slamming and sharp pitching, is used as data of wave sea conditions for the following calculation. According to Fig. 2, we can solve the long-term extreme value of these sea conditions after determining the extreme sea condition data of green water, slamming and sharp pitching.

5) Long-term extreme value of sea conditions.

After determining the data of extreme sea conditions, it is also necessary to use the probability data of each short-term sea condition in the scatter diagram to fit the expression of long-term extreme value of the sea condition. The distribution function of a single short-term sea condition is the joint probability distribution function of the significant wave height $H_s$ and the corresponding spectral peak period $T_p$:

$$F\left(H_s, T_p\right) = F\left(H_s\right) F\left(T_p \middle| H_s\right) \tag{9}$$

In the formula, the relationship between $T_p$ and average zero-crossing period $T_z$ is $T_p = 1.41 T_z$. The marginal probability distribution function of significant wave height conforms to the three-parameter Weibull distribution

$$F\left(H_s\right) = 1 - \mathrm{e}^{-\left(\frac{H_s - \gamma}{\alpha}\right)^{\beta}} \tag{10}$$

The conditional probability distribution function of $T_p$ conforms to the lognormal distribution

$$F\left(T_p \middle| H_s\right) = \Phi\left(\frac{\ln T_p - \mu}{\sigma}\right) \tag{11}$$

According to the probability theory, using probability of extreme wave sea condition data under this sea condition in the wave scatter diagram of the North Atlantic, parameters $\alpha$, $\beta$, $\gamma$ in Formula (10) of $H_s$ probability distribution function, as well as $\mu$ and $\sigma$ in Formula (11) of $T_p$ probability distribution function were fitted.

In order to calculate the long-term distribution of extreme wave bending moment under extreme wave sea conditions, we need to establish a relationship between the long-term extreme value of significant wave height $H_{s,max}$ as well as its corresponding $T_p$ and the standard normal random variables.

Assuming that each short-term wave sea condition was independent of each other, the long-term extreme value distribution function of the significant wave height was obtained according to sequence statistics

$$F_{H_{s,max}}\left(H_s\right) = F\left(H_s\right)^N \tag{12}$$

In the formula, $N$ was the number of short-term sea conditions in 3 h, and the distribution was normalized, then there is

$$F_{H_{s,max}}\left(H_s\right) = \Phi(B) \tag{13}$$

That is, $\left\{1 - \mathrm{e}^{-\left(\frac{H_{s,max} - \gamma}{\alpha}\right)^{\beta}}\right\}^N = \Phi(B)$. Through Formula (13), there is

$$H_{s,max}=\gamma+\alpha\left[-\ln\left(1-\exp\left(\frac{1}{N}\ln\Phi(B)\right)\right)\right]^{\frac{1}{\beta}} \quad (14)$$

Similarly, the spectral peak period was normalized, namely, $F\left(T_p\middle|H_s\right)=\Phi(C)$, then expression of spectral peak period $\beta=1.937$ under the extreme significant wave height $H_{s,max}$ was obtained:

$$T_p=e^{\mu(H_{s,max})+\sigma(H_{s,max})C} \quad (15)$$

Formulas (14) and (15) are the expressions of long-term extreme sea conditions. The long-term extreme significant wave height $H_{s,max}$ is a random variable related with standard normal random variable $B$, and the corresponding $T_p$ is a random variable related with standard normal random variable $C$.

## 1.2 The extreme value of the long-term wave bending moment considering the extreme wave sea conditions

After the long-term extreme value of sea conditions was determined, it is necessary to use the eigenvalues of the long-term wave bending moment distribution considering green water, slamming and sharp pitching, to solve the long-term extreme value of wave bending moment.

The distribution eigenvalue of long-term wave bending moment includes variance $\sigma^2_{s,max}$ and average zero-crossing period $T_r$ of wave bending moment.

1) The variance of long-term wave bending moment considering green water, slamming and sharp pitching.

Using the North Atlantic scatter diagram under extreme wave conditions, combined with the two-parameter Pearson-Moscovici spectrum (P-M spectrum) and frequency response transfer function of wave bending moment, the significant value of wave bending moment of each short-term sea condition was calculated, and then the variance of wave bending moment under extreme wave sea conditions was calculated according to the following steps:

(1) The significant value of the wave bending moment of each short-term sea condition was divided by the significant wave height to obtain the significant value of the short-term wave bending moment $M_s/H_s$ under the unit significant wave height.

(2) Short-term sea conditions with the same average zero-crossing period were seen as one group. Assuming that the probability of occurrence of this group was 1, and according to the probability of occurrence of each short-term sea condition in the scatter diagram under extreme sea conditions, the probability of occurrence in the group was calculated, which was used as the weighted value to solve the weighted sum of significant value $M_s/H_s$ of short-term wave bending moment under the unit significant wave height within the group.

(3) The expression of $M_s$ was obtained by using the relational expression between $M_s/H_s$ and $T_z$

$$M_s(H_s,T_z)=$$
$$H_s\cdot(a_0+a_1T_z+a_2T_z^2+a_3T_z^3+a_4T_z^4+a_5T_z^5) \quad (16)$$

For any short-term sea condition, the variance of the wave bending moment can be obtained according to the relationship with the above root mean square of significant value of the wave bending moment

$$\sigma_s^2=\left(\frac{M_s(H_s,T_z)}{2}\right)^2 \quad (17)$$

The long-term extreme sea condition data $H_{s,max}$ and $T_p$ in Formulas (14) and (15) considering green water, slamming and sharp pitching were substituted into Formula (17), variance $\sigma^2_{s,max}$ of long-term extreme wave bending moment under the extreme wave sea conditions can be obtained.

2) The average zero-crossing period of wave bending moment under extreme wave sea conditions.

(1) In the North Atlantic scatter diagram under extreme sea conditions, the average zero-crossing period of wave bending moment of each $T_z$ under the unit significant wave height was $T_r$;

(2) The relationship between the average zero-crossing period $T_r$ of wave bending moment and the average zero-crossing period $T_z$ of wave was established:

$$T_r=b_0+b_1T_z+b_2T_z^2+b_3T_z^3 \quad (18)$$

The spectral peak period $T_p$ ($T_p=1.41T_z$) of Formula (15) was substituted into Formula (18), then the zero-crossing period $T_r$ of the wave bending moment can be obtained.

(3) Long-term extreme value distribution of wave bending moment under extreme wave conditions of green water, slamming and sharp pitching.

Based on the above-mentioned steps, the long-term extreme value of sea conditions and the distribution eigenvalues of long-term wave bending moment were obtained. According to the thought in Fig. 2, the extreme value of long-term wave bending moment under extreme wave sea conditions can be solved.

Firstly, the extreme value distribution of long-term wave bending moment was given[3]:

$$F_{M_w}(m) = \exp(-v(m)D) \tag{19}$$

$$v(m) = v_0 \exp\left(-\frac{m^2}{2\sigma_s^2}\right) \tag{20}$$

In the formula: $v(m)$ is the probability that the peak value of the wave bending moment passes through the extreme value $m$; $D$ is the duration of short-term sea condition, set to 3 h; $v_0$ is the average zero-crossing rate of wave bending moment, and $v_0 = 1/T_r$; $\sigma_s$ is the standard deviation of the long-term wave bending moment.

Standard normalization was carried out for distribution: $F_{M_w}(m) = \Phi(A)$, and the relationship between the extreme value of wave bending moment and the standard normal random variable A can be obtained

$$M_W = \left[-2\sigma_s^2 \ln\left\{-\frac{T_r}{D}\ln \Phi(A)\right\}\right]^{1/2} \tag{21}$$

Using variance $\sigma_{s,max}^2$ of the long-term extreme wave bending moment determined by Formulas (17) and (18) and the corresponding zero-crossing period $T_r$ of wave bending moment under extreme wave conditions, extreme long-term wave bending moment $M_{UM}$ can be obtained considering the extreme sea conditions of green water, slamming and sharp pitching. According to the above normal transformation, we can see that the extreme value of extreme wave bending moment can be expressed as the expression of 3 normal random variables *A*, *B* and *C* after all variables were substituted

$$M_{UW} = M_{UW}(A, B, C) \tag{22}$$

In the formula, random variables *A*, *B* and *C* respectively represent the impact of wave spectral peak period $T_p$ and significant value $M_s$ of short-term wave bending moment that correspond to the long-term extreme value $H_{s,max}$ of significant wave height on the extreme value of wave bending moment under extreme wave sea conditions. It can be seen that the long-term extreme value of wave bending moment under extreme wave sea conditions is a complex random variable when the 3 random variables were substituted into Formula (22).

## 2 Reliability method

From the above analysis, we know that the long-term extreme value of wave bending moment considering extreme sea conditions of green water, slamming and sharp pitching is a complex random variable constituted by 3 standard normal random variables, so the distribution of limit state function is more complex, and we need to investigate the respective characteristics of different calculation methods of reliability, so as to select the efficient and reasonable calculation method of longitudinal ultimate strength reliability of ships.

In the DNV guide for reliability calculation of ship structure[3], the limit state function of longitudinal ultimate strength of ship is defined as

$$Z = M - M_{UW} - M_{SW} = 0 \tag{23}$$

In the formula: $M$ is the longitudinal ultimate strength of the structure; $M_{SW}$ is the long-term maximum value of still water bending moment.

In considering the extreme sea conditions such as slamming, green water and sharp pitching, factors needing to be considered in the reliability analysis of longitudinal ultimate strength of ships did not change, therefore, the reliability calculation of longitudinal ultimate strength under extreme wave conditions can still be expressed in the form of limit state function in Formula (23). The long-term maximum value $M_{SW}$ of still water bending moment did not change, and only $M$ and $M_{UW}$ changed.

In the current calculation of reliability, $M$ is generally processed as the normal random variable[10], and $M_{SW}$ is a determined quantity. As for the extreme wave bending moment in considering green water, slamming and sharp pitching, $M_{UM}(A, B, C)$ is a random variable related to 3 normal random variables, which makes the distribution of $M_{UW}$ more complicated. Under the precondition of considering extreme load and ultimate strength, it is necessary to find a new method of reliability calculation that has no requirement for the distribution of basic random variables and the distribution of performance function. Table 2 shows the characteristics of commonly used reliability methods[2, 11].

As can be seen from Table 2, when the limit state function was explicit, we can directly calculate the reliability index and failure probability by the first-order second-moment (FOSM) method, but only the center point method has no requirement for the distribution of basic random variables in the FOSM method; Monte-Carlo method has relatively high precision, but it requires a large number of samplings, thus this method is generally used for verifying the accuracy of other methods; the adaptive importance sampling method is more efficient than Monte-Carlo method, but the programming is difficult; the response surface method is used for handling the system that limits state function cannot ex-

**Table 2  The characteristics of common reliability calculation methods**

| Reliability analysis method | | Advantage | Disadvantage |
|---|---|---|---|
| First-order second-moment method | Center point method | The calculation process is simple, and the failure probability of structure can be obtained by a single calculation, no requirement for the distribution of basic variables | The limit state function must be explicit. The failure probability deviation is large since the expansion points are average points which are usually located in the reliable region. Different reliability indices will be obtained when different limit state functions are adopted |
| | Improved first-order second-moment method | The expansion points are located on the failure surface, which overcomes the shortcomings of the mean first-order second-moment method, i.e. large error | The limit state function must be explicit. The failure probability has certain accuracy only when the basic variables conform to the normal distribution and have a linear safety margin equation |
| | JC method | The random variables are normalized, and this method is suitable for general engineering structures | The limit state function must be explicit, and the equivalent normalization process is difficult |
| Monte-Carlo method | | There is no requirement for distribution of basic variables, and Monte-Carlo method is a relatively accurate method in the current reliability analysis methods, which is usually used to determine whether the test results are correct or not | It requires a huge number of samplings. Generally, the calculation time is relatively long |
| Adaptive importance sampling method | | The adaptive importance sampling method is improved on the basis of Monte-Carlo method, and the accuracy is guaranteed | The theoretical basis is obscure, which is usually difficult to understand, and the process of independent programming is difficult |
| Response surface method | | Response surface method is used to solve the structure that the performance function cannot express explicitly | In the process of calculation, it is necessary to construct the response surface continuously, and then carry out iterative calculation until the result meets the error requirement, so the calculation process is relatively complex |
| Random finite element method | | The method is relatively advanced and organically combined with the finite element analysis method | For each sample, a finite element analysis is needed, and the consumption of manpower and time is huge |

press, which requires multiple iterations and construction of response surface, hence the calculation process is complex; random finite element method requires finite element analysis for each sample, hence great consumption of manpower and time. In the case that limit state function is in the form of Formula (20), and the basic random variables such as structural strength variables and load variables are subject to complex distribution, Monte-Carlo method and center point method were selected preliminarily, and further analysis was conducted.

The efficiency and accuracy of the center point method and Monte-Carlo method were calculated by using the self-test examples in the reliability analysis software NESSUS, and the results are shown in Table 3.

**Table 3  Results by Monte-Carlo method and FOSM**

| Reliability method | Sampling times $N$ | Failure probability | Time consumption of calculation/s |
|---|---|---|---|
| Center point method | | 0.029 90 | 0.36 |
| Monte-Carlo method | 1 000 | 0.030 00 | 0.50 |
| | 100 000 | 0.029 30 | 16.04 |
| | 500 000 | 0.030 25 | 73.41 |
| | 1 000 000 | 0.030 17 | 139.90 |

As a verification index of calculation accuracy, Monte-Carlo method needs enough times of sampling to support the accuracy. It is generally believed that the sampling times must meet the requirements of $N \geqslant 100/P_f$ ( $P_f$ is failure probability)[2]. If the Monte-Carlo method is used in the examples, to ensure accuracy, the sampling times must be greater than 3 333. From Table 3 we know that this will make the computation time increase exponentially, while the center point method not only is feasible, but also can still meet the engineering requirements of accuracy compared with the Monte-Carlo method. Therefore, comprehensively considering the efficiency and accuracy, it is suggested that the center point method be used.

## 3 Example

The extreme wave bending moment calculation method proposed above, considering extreme wave sea conditions such as green water, slamming and sharp pitching, as well as the reliability calculation method of longitudinal ultimate strength of ships was applied to examples, to calculate the reliability within the year, so as to verify the feasibility and rationality of the above methods. For this purpose, a ship was constructed as an example in this paper. Fig. 5 and Table 4 are schematic and principal dimension parameters of the model, thus solving the structural strength and still water bending moment. Then according to the above methods, the annual extreme values of ship's wave bending moment under extreme wave conditions and the reliability of longitudinal ultimate strength of ships under extreme wave conditions were calculated.

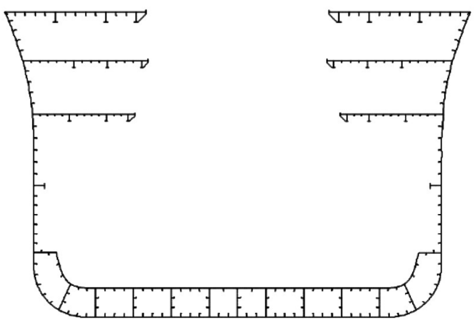

Fig.5 Cross-sectional view

**Table 4 Principal dimensions of model**

| Principal dimension | Value |
|---|---|
| Length $L_w$ / m | 148 |
| Breadth $B$ / m | 21.25 |
| Height $D$ / m | 13.9 |
| Draught $T$ / m | 4.9 |
| Displacement $\Delta$ / t | 8 238 |
| Block coefficient $C_b$ | 0.614 |

In the reliability analysis of the ultimate strength of ships, ultimate strength influence parameters and ultimate strength itself are random variables. Plate thickness, yield stress and stiffener thickness were selected as the random variable parameters affecting the ultimate strength, and with reference to the suggestions of parameters in related references, using the nonlinear finite element method for the calculation of the ultimate strength of ships in the references, the ultimate strength values under the influence of different random variable parameters were obtained. Finally, the statistical eigenvalues of the ultimate strength were obtained by the Rosenbluth method [12].

In a case where there are detailed data of ship (such as the loading manual of ships), the extreme value of still water bending moment can be directly calculated, but actual examples lack of this kind of data, so we made reasonable forecast of the extreme value of still water bending moment of ship with reference to the method in the DNV specification, and the annual maximum still water bending moment of examples can be obtained. Through the above calculation methods, the annual maximum value of ultimate strength and still water bending moment of examples can be obtained, as shown in Table 5.

**Table 5 Ultimate strength and still water load**

| Statistical characteristics | Mean/(N·m) | Variance | Distribution pattern |
|---|---|---|---|
| Ultimate strength | $1.899 \times 10^9$ | $1.822\,5 \times 10^{16}$ | Gaussian distribution |
| Still water bending moment | $2.850 \times 10^8$ | | Constant |

## 3.1 Extreme load calculation

For the large ships in the overall sea area, the extreme load under extreme sea conditions mainly refers to the wave bending moment. Therefore, with reference to the steps in Fig. 2, first, navigation boundaries of green water, slamming and sharp pitching are used to solve the navigational limit wave height, short-term data of sea conditions in the wave scatter diagram were defined in turn for the calculation of wave bending moment, and according to the data of sea conditions, the extreme value distribution of sea conditions were obtained. Then, the short-term wave bending moment was calculated using data of sea conditions through the spectral analysis method and hereby the long-term distribution eigenvalue of wave bending moment was obtained. Finally, the eigenvalue of the long-term extreme value distribution of wave bending moment and the extreme value distribution of sea conditions were substituted into the formula of long-term extreme value of wave bending moment, thus solving long-term extreme value of wave bending moment under extreme wave sea conditions.

1) Determination of extreme sea conditions considering green water, slamming and sharp pitching.

According to the calculation steps in Fig. 2, first we used the navigational boundary value selected to solve the navigational limit wave height under extreme sea conditions of green water, slamming and sharp pitching, and thus determine the extreme sea condition data under extreme wave sea conditions. According to the proposal of section 1.1, the fourth group of data in Table 1 of navigational boundary reference value was selected as the navigational boundary value, and then the relative motion response function of the calculated point was obtained by the numerical method. $\bar{R}_1$, $\bar{R}_2$ and $\bar{R}_3$ were obtained by the spectral analysis method, and the standard deviation of the above motion was substituted into wave height boundary Formulas (4), (7) and (8), wave height boundary of green water $H_{m_1}$, that of slamming $H_{m_2}$ and that of sharp pitching $H_{m_3}$ were obtained, as shown in Table 6. The above-mentioned relative motion response functions were calculated by the SESAM software of DNV, and the calculated wet surface model is shown in Fig. 6.

**Table 6 Boundary of significant wave height under extreme wave load**

| Extreme wave sea condition | Wave height boundary/m |
|---|---|
| Green water | 3.634 |
| Slamming of the bottom | 2.908 |
| Sharp pitching | 2.116 |

Fig.6 Wet surface of the hull

According to the results in Table 6 and the proposal of section 1.1, the North Atlantic wave scatter diagram of $H_s \geqslant 3.634$ m was selected as data of extreme wave sea condition. Using the probability data of short-term sea conditions in the scatter diagram, and according to the method of probability theory, parameters $\alpha$, $\beta$, $\gamma$, $\mu$ and $\sigma$ in Formulas (10) and (11) were fitted, and the annual extreme values $H_{s,max}$ and $T_p$ of sea condition determined by Formulas (14) and (15) were obtained:

$$H_{s,max} = 3.289 + 1.96[-\ln(1 - e^{(1/N \ln \Phi(B))})]^{0.789} \quad (24)$$

$$T_p = e^{\mu(H_{s,max}) + \sigma(H_{s,max})C} \quad (25)$$

where

$$\mu(H_{s,max}) = 1.187 + 0.833 H_{s,max}^{0.242} \quad (26)$$

$$\sigma^2(H_{s,max}) = 0.117\,7 e^{-0.312 H_{s,max}} + 0.005\,41 e^{-0.041 H_{s,max}} \quad (27)$$

2) The eigenvalue of long-term wave bending moment distribution.

According to the thoughts in Fig. 2, through the steps in section 1.2, the significant values of each short-term wave bending moment under extreme wave conditions of green water, slamming and sharp pitching were first calculated. Combined with the probability data of each short-term sea condition in the wave scatter diagram under extreme wave sea conditions, eigenvalues of long-term distribution of wave bending moment were fitted using Formulas (16) and (17), such as variance of wave bending moment $\sigma_{s,max}^2$ and average zero-crossing period of wave bending moment $T_r$:

$$M_s = H_{s,max} \cdot (0.014 \times 10^4 T_z^5 - 1.494 \times 10^4 T_z^4 + 55.121 \times 10^4 T_z^3 - 917.194 \times 10^4 T_z^2 + 6\,687.288 \times 10^4 T_z - 12\,866.390 \times 10^4) \quad (28)$$

$$\sigma_{s,max}^2 = \left( \frac{M_s(H_{s,max}, T_z)}{2} \right)^2 \quad (29)$$

$$T_r = 0.001\,8 \times T_z^3 - 0.087\,9 \times T_z^2 + 1.524\,7 \times T_z + 1.525\,6 \quad (30)$$

3) The annual extreme value of extreme wave bending moment considering green water, slamming and sharp pitching.

Finally, $\sigma_{s,max}^2$ and $T_r$ were substituted into the annual extreme value Formula (21) of wave bending moment, then the annual extreme value of wave bending moment under extreme wave conditions was obtained. The calculation result was compared with the annual extreme value of wave bending moment without considering the extreme sea conditions calculated in the DNV guide for ship structural reliability [4], as shown in Table 7.

**Table 7 Prediction of long-term wave load**

| Statistical characteristics | Mean/(N·m) | Variance |
|---|---|---|
| Wave bending moment | $8.720\,7 \times 10^8$ | $1.284\,9 \times 10^{17}$ |
| Wave bending moment under extreme wave sea conditions | $9.721\,4 \times 10^8$ | $1.358\,7 \times 10^{17}$ |

Compared with the calculated results of the long-term extreme wave bending moment in the DNV rules, the average extreme value of wave bending moment considering extreme wave conditions of green water, slamming and sharp pitching was increased by 11.5%. This is due to that the original wave load calculation method does not involve the extreme sea condition which has large influence on the longitudinal bending moment of ship, thus causing the lower calculated value of wave bending moment. Chen[13] recorded the experimental results of a ship under the condition of head sea in the South China Sea, and the experimental value of mean wave bending moment was 9.82% larger than that without considering slamming, green water and other factors. Although the navigational conditions of the experimental ship had some differences with the calculation conditions of the example, they had a certain correlation with the analysis and calculation of the annual extreme prediction results of bending moment in this paper, which verifies the rationality of this method to a certain extent.

## 3.2 Reliability calculation

The long-term extreme value of wave bending moment given in section 3.1 and data of the structural ultimate bending moment and still water bending moment in Table 5 were applied to the center point method, and the accuracy of the index was verified by the Monte-Carlo method. The failure probability and reliability index of the examples were obtained using the reliability calculation software NESSUS, as shown in Table 8.

**Table 8 Reliability calculation of the model**

| Calculation method | Failure probability $P_f$ | Reliability index $\beta$ |
|---|---|---|
| Monte-Carlo method | 0.026 325 00 | 1.937 779 |
| Center point method | 0.026 375 11 | 1.937 000 |

The calculated results of Table 8 show that, the center point method applies to the calculation of longitudinal ultimate strength reliability of ships considering the extreme sea conditions of green water, slamming and sharp pitching, and compared to the results of Monte-Carlo method, its accuracy meets the requirement of engineering calculation as well. Therefore, combined with the discussion on reliability method in section 2.1, the center point method can be used as the reliability calculation method of longitudinal ultimate strength of ships under extreme wave conditions.

# 4 Conclusions

Aiming at the research on the reliability of longitudinal ultimate strength of ship under extreme load conditions, the influence factors of the ultimate bearing capacity of structure are taken into overall consideration and further refined. Considering the wave loads considering the extreme sea conditions such as green water, slamming and sharp pitching, the reliability calculation method of longitudinal ultimate strength of ships under extreme load conditions was summarized. The following conclusions are obtained through the research:

1) From the perspective of navigational wave height boundary of ship, we proposed a method for calculating wave loads under extreme wave sea conditions, that is, the engineering calculation method of the wave bending moment distribution considering green water, hull bottom slamming, and sharp pitching of ships. Compared with the general long-term calculation method of extreme wave bending moment, the extreme load obtained was 11.5% higher. This is due to that the original wave load calculation method does not involve the extreme wave conditions which have great influence on the ship's longitudinal bending moment. Compared with the experimental results of ship in the South China Sea, it can be seen that the method can more truly reflect the sea conditions encountered by ship to a certain extent.

2) In the reliability analysis of longitudinal ultimate strength of ships under extreme load conditions, the basic random variables have complex distribution pattern, therefore the limit state function does not conforms to the general distribution, and it is difficult to obtain the statistical characteristics of basic random variables that affect extreme load and the structural ultimate strength of ships. On this basis, comparing the calculation method of reliability, and referring to the calculation results of examples and model, the center point method was selected as the reliability calculation method of longitudinal ultimate strength of ships under extreme load conditions, which has more extensive applicability, less requirement for statistics, no requirement for the distribution pattern of performance function, and basically the same accuracy with Monte-Carlo method.

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# 舵系统流激振动影响因素及规律的理论与试验研究

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**摘 要：**流激舵系统引起的振动对水下航行体隐蔽性产生较大影响。为深入研究其振动特性，根据舵系统的结构组成进行简化，建立系统二元线性颤振数学模型，确定低速颤振的产生条件，并获得低速颤振的主要影响因素和作用规律。此外，在重力式水洞中开展舵模型流激振动试验，重点研究了支撑刚度、扭转刚度、质心和刚心位置等参数变化对舵模型流激振动的影响。结果表明：在流体载荷激励下，舵系统结构设计对流激振动特性有较大影响，通过对升沉运动与扭转运动频率之比、结构质量与附加质量之比、刚心、质心与弦中心的相对位置等参数进行匹配设计，能够有效抑制舵系统流激振动。

**关键词：**舵系统；流激振动；低速颤振；水洞

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# 极端海况下船舶总纵极限强度可靠性计算方法

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**摘 要：**船舶全海域大型化是一个发展趋势，因此船舶总纵极限强度可靠性计算中需要将极端波浪的影响参数考虑在内。一般的载荷计算方法并没有考虑极端海况中出现的特殊波浪载荷的影响；另外对于可靠性分析，极端载荷是更为复杂的随机变量，一般的船舶可靠性计算方法因为局限于某种特定分布，可能出现无法适用的问题。选取极端海况中上浪、砰击和大幅纵摇等对船舶总纵波浪弯矩有较大影响的因素，从航行界限的角度出发，将这些因素引入极端波浪弯矩的计算中，所得极端波浪海况下的波浪弯矩极值数据比常规波浪弯矩极值更大。参考实验数据表明，考虑极端波浪海况的波浪弯矩计算方法能在一定程度上更加真实地反映船舶所受波浪载荷；其次通过考察不同可靠性计算方法的特点，利用实例计算，给出极端海况下船舶总纵极限强度可靠性计算方法的选取建议。

**关键词：**极端波浪海况；极端载荷；总纵极限强度；可靠性计算