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Study on acoustic vibration similarity law of complex stiffened cone-cylinder shell

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Abstract: [**Objective**] Due to the difficulty of accurately converting the experimental results of acoustic vibration from scale models of complex stiffened combined shells into that of prototypes, the acoustic vibration similarity law of the shell is studied, so as to provide a basis for experimental research on the scale model based acoustic vibration test of such underwater structures. [**Methods**] First, a complex stiffened cone-cylinder shell model and its scale model are constructed by using shell elements for reinforcement simulation. Next, on the basis of the finite element method-boundary element method (FEM-BEM), the acoustic vibration response of the combined shell model is calculated. Based on the model experiment, this paper verifies the accuracy of the calculated vibration response of the combined shell by the FEM-BEM. Finally, the acoustic vibration similarity law of the complex stiffened cone-cylinder shell is studied systematically. [**Results**] The modal frequency of the combined shell is inversely proportional to the scale ratio under the same material parameters, boundary conditions, and exciting force, while the vibration modal in the corresponding frequency is the same. Under the same exciting force, the vibration response of the combined shell is also inversely proportional to the scale ratio, and the acoustic pressure is inversely proportional to the product of the scale ratio and field point distance of the shell. In addition, the acoustic radiation efficiency and acoustic pressure directivity of the underwater scale model and prototype are the same. [**Conclusions**] The stiffened cone-cylinder shell represents good acoustic vibration similarity laws under similar conditions, and the requirements of studying the acoustic vibration characteristics of the prototype of the combined shell by scale model experiment under similar conditions are met.

Key words: cone-cylinder shell; similarity law; finite element method-boundary element method (FEM-BEM)

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0 Introduction

With the development of sonar detection, the acoustic stealthiness of submarines directly affects their combat capabilities and vitality. As vibration noise (hereinafter referred to as acoustic vibration) directly affects acoustic stealthiness, it has become one of the key performance indexes of submarine design [1]. Submarines need to work underwater for a long time. In such a case, over-high acoustic vibration will not only jeopardize the physical and mental health of the crew but also damage the ma-

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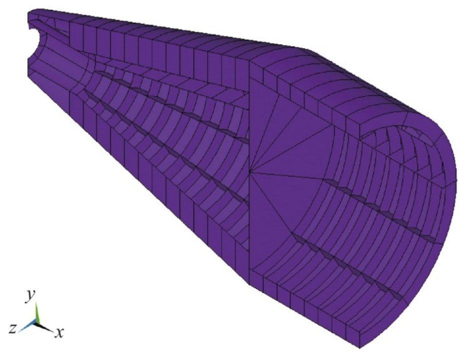
rine ecological environment to some extent. Therefore, acoustic vibration control is an important part of submarine design and research. However, acoustic vibration tests of real submarines bring many problems to the research, such as large physical size, high test cost, long test cycle, and high difficulty. As a result, scholars both in China and abroad prefer to use scale models for related tests and then convert the data obtained from these models into those for real submarines. Thus, it is necessary to study the similarity law of scale models. At present, some achievements have been made in studying the acoustic vibration similarity of shells. However, as to relatively complex combined shells (such as real submarines with complex structures), acoustic vibration test results of scale models fail to predict acoustic vibration characteristics of prototypes accurately.

Global scholars have studied the structural acoustic vibration similarity to some extent. In view of a stiffened cylindrical shell, Li et al. [2] analyzed the influence of different similarity conditions on the structural acoustic vibration similarity and summarized similarity mechanisms of vibration and acoustic radiation of underwater structures. Coutinho et al.[3] summarized similarity theories in structural engineering (including theories based on dimensional analysis, differential equations, and energy method), which provides support for studying structural similarity. With both finite element analysis (FEA) and test, Balawi [4] studied vibration similarity ratios of shells. The results show that it is feasible to study vibration similarity laws of shells by the finite element method (FEM). Yu et al. [5] studied the similarity of the coupled acoustic vibration response of underwater elastic structures. They established underwater vibration equations of the structures by using the finite element method-boundary element method (FEM-BEM) and deduced similarity ratios of underwater acoustic vibration characteristics. Moreover, they verified the accuracy of the similarity theory experimentally. On the basis of summarizing different methods of similarity analysis, Bai [6] deduced the similarity theory by dimensional analysis and then verified the structural vibration similarity by finite-element simulation and calculation. Based on stiffened cylindrical shells that are completely similar, Petrone et al. [7] studied the vibration similarity in the case of structural distortion by FEM. Their study indicates that positive prediction results can also be obtained in view of distorted structures. In addition, based on the experimental similarity principle, Shi et al. [8] obtained similarity ratios of the modal and vibration responses of completely similar structural models by dimensional analysis. Rong et al. [9] established a similarity ratio of structural scale models by dimensional analysis and verified it by FEM, which provides a theoretical basis for acoustic vibration prediction and tests of scale structures.

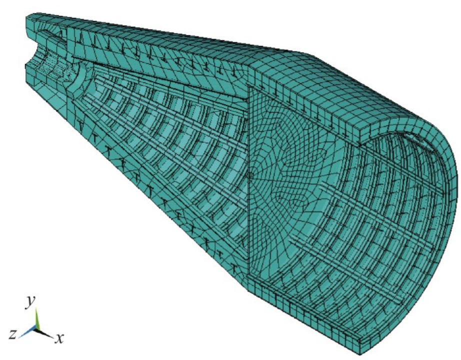
In conclusion, current research on acoustic vibration similarity of shells mainly focuses on shells with simple structures, and the similarity of cylindrical or conical shells is analyzed. However, the similarity of complex structures (such as stiffened combined shells) under fluid coupling is not studied sufficiently. For this reason, this paper established a complex stiffened cone-cylinder shell and its scale model by using shell elements for reinforcement simulation and calculated the structural modal and vibration responses in the air based on a model constructed in ANSYS. Then, the finite element model and modal data were imported into the acoustic calculation software Virtual.Lab Acoustics to calculate underwater coupled modal and vibration responses by BEM. Finally, by model test, the paper verified the accuracy of vibration response of the complex combined shell by the FEM-BEM, as well as the acoustic radiation efficiency and acoustic directivity of the underwater combined shell. Moreover, the acoustic vibration similarity law of the complex combined shell was studied.

1 Modeling of complex stiffened cone-cylinder shell

In order to study the acoustic vibration similarity law of a complex stiffened cone-cylinder shell, this paper established a model of such a structure, which consists of geometric models of a shell, a transverse bulkhead, longitudinal-rib webs, and ring-rib webs, as shown in Fig. 1. In the model, a coordinate system was set up with its origin at the center of the small end of the cone, its x-axis in the axial direction, its y-axis in the vertical direction, and its z-axis in the horizontal direction. Parameters of the shell prototype were as follows: shell length of 12 m (cone length of 8 m and cylinder length of 4 m), cylinder radius of 3.5 m, shell thickness of 0.04 m, cone small-end radius of 1 m, and cone large-end radius of 3.5 m. In addition, the ring-rib height of the cone was 0.4 m; the radius of the tail bearing support ring was 0.7 m; the thickness of the inner transverse bulkhead and longitudinal-rib webs was 0.025 m. In the cylinder, the heights of ring ribs and longitudinal ribs were 0.4 m and 0.5 m, respectively, and ring ribs were uniformly distributed, with a spacing of 0.5 m. The 1∶4 scale model was completely similar to the prototype.



(a) Geometric structure



(b) Finite element model

Fig.1 Geometric structure and finite element model of stiffened cone-cylinder shell

Material properties of the shell were as follows: Young's modulus of 2.1×10^{11} N/m^2, Poisson's ratio of 0.3, the density of 7 800 kg/m^3, and the damping ratio of 0.003. In the constructed finite element model, the cone-cylinder shell and webs of ring and longitudinal ribs were simulated by SHELL 181 shell elements, while panels of ring and longitudinal ribs were simulated by BEAM 188 beam elements. According to the wavelength theory of bending waves of shell vibration (namely, six elements in one wavelength), the element size of the finite element model was calculated. For the prototype, the calculated frequency was 10−100 Hz, and the element size was 0.138 m. For the 1:4 scale model, the calculated frequency was 40−400 Hz, and the element size was reduced according to the scale.

2 Derivation of acoustic vibration similarity theory of shell

2.1 Modal similarity of shell

According to the thin shell theory, the finite element equation of the shell under undamped free vibration [10] is as follows:

$$\left(\boldsymbol{K}-\omega^2\boldsymbol{M}\right)\boldsymbol{U}=0 \tag{1}$$

where $\boldsymbol{K}$ is a total stiffness matrix; $\boldsymbol{M}$ is a total mass matrix; $\boldsymbol{U}$ is a displacement column matrix; ω is the natural frequency.

Suppose that the ratio between physical quantities of the same type in both the scale model and the prototype is denoted as λ; φ' represents a physical quantity of the scale model (physical quantities with the superscript " ′ " hereinafter all represent those of the scale model), and φ represents a physical quantity of the prototype, then the following formula is obtained.

$$\lambda_\varphi=\frac{\varphi'}{\varphi} \tag{2}$$

If the radius r, length l, and thickness h of the prototype are reduced to those of the scale model according to the same proportion, this paper has $\lambda_r = \lambda_l = \lambda_h$. Then, the relationship between the mass and stiffness matrixes of the scale model and those of the prototype is expressed as follows:

$$\lambda_{\boldsymbol{M}}=\lambda_l^3\lambda_\rho,\quad \lambda_{\boldsymbol{K}}=\lambda_E\lambda_l \tag{3}$$

where λ_l is the ratio of the length of the scale model to that of the prototype; λ_ρ is the ratio of the density of the scale model to that of the prototype; λ_E is the ratio of Young's modulus of the scale model to that of the prototype.

The characteristic equation of the scale model is given by

$$\left(\boldsymbol{K}'-\omega'^2\boldsymbol{M}'\right)\boldsymbol{U}'=0 \tag{4}$$

By substituting Formula (3) into Formula (4) and then comparing the new formula with Formula (1), this paper can obtain a similarity ratio shown in Formula (5).

$$\lambda_\omega=\frac{1}{\lambda_l}\sqrt{\frac{\lambda_E}{\lambda_\rho}} \tag{5}$$

where λ_ω is the ratio of the natural frequency of the scale model to that of the prototype.

The scale model and the prototype have similar geometry and the same boundary conditions. In addition, their characteristic equations are similar. Therefore, their vibration modal is the same, while their natural frequencies are inversely proportional to geometric sizes.

When the shell is immersed in water, it is assumed that the mass of water is uniformly distributed on the surface of the shell, and the mass of the added water and that of the shell are superimposed to produce a new mass matrix. In addition, radiation damping produced by the underwater shell is superimposed with the structural damping of the shell to produce a new damping matrix. On this basis, the wet modal frequencies and vibration modal of the underwater shell can be obtained by solving the vibration equation. The loading force of fluids [10] can be expressed as

$$\boldsymbol{f}_{\mathrm{a}}=\frac{1}{\rho_{\mathrm{a}}}\boldsymbol{Sp}=-\left[\bar{\boldsymbol{R}}+\mathrm{i}\omega\bar{\boldsymbol{M}}\right]\dot{\boldsymbol{U}} \tag{6}$$

where $\boldsymbol{p}$ is a column matrix of fluid pressure; ρ_{a} is the density of fluid; $\bar{\boldsymbol{R}}$ is a matrix of acoustic radiation damping; $\bar{\boldsymbol{M}}$ is an equivalent mass matrix of fluid; $\boldsymbol{S}$ is an element coupling matrix; $\dot{\boldsymbol{U}}$ is the vibration velocity ($\boldsymbol{U}$ is the vibration displacement).

The finite element equation of the underwater shell under free vibration is given by

$$\left[\boldsymbol{K}-\omega^2(\boldsymbol{M}+\bar{\boldsymbol{M}})\right]\boldsymbol{U}=0 \tag{7}$$

When the scale model and the prototype are of the same material, and the shell is immersed in the same fluid, the similarity ratio of the wet modal frequency of the underwater shell is as follows:

$$\lambda_{\omega_{\mathrm{f}}}=\frac{1}{\lambda_l} \tag{8}$$

where $\lambda_{\omega_{\mathrm{f}}}$ is the ratio of the natural frequency of the underwater scale model to that of the underwater prototype.

From Formula (8), the shell has consistent similarity law both in water and the air in terms of the natural frequency. In other words, the natural frequency is inversely proportional to geometric size.

2.2 Similarity of vibration response of shell

The finite element equation of the shell under excited vibration is given by

$$\boldsymbol{M}\ddot{\boldsymbol{U}}+\boldsymbol{C}\dot{\boldsymbol{U}}+\boldsymbol{K}\boldsymbol{U}=\boldsymbol{F} \tag{9}$$

where $\boldsymbol{C}$ is a total damping matrix; $\boldsymbol{F}$ is the exciting force; $\ddot{\boldsymbol{U}}$ is the vibration acceleration. According to the above analysis, the finite element equation of the scale model under excited vibration can be expressed as follows.

$$\boldsymbol{M}'\ddot{\boldsymbol{U}}'+\boldsymbol{C}'\dot{\boldsymbol{U}}'+\boldsymbol{K}'\boldsymbol{U}'=\boldsymbol{F}' \tag{10}$$

When the scale model and the prototype are under the same point exciting force, their element nodal force satisfies the following formula:

$$\lambda_{\boldsymbol{F}}=\lambda_{\boldsymbol{F}'} \tag{11}$$

When the scale model is completely similar to the prototype, the similarity ratios of total mass and total stiffness matrixes between the scale model and the prototype are expressed as follows, respectively.

$$\begin{cases}\lambda_{\boldsymbol{M}}=\lambda_\rho\lambda_l^{\,3}\\ \lambda_{\boldsymbol{K}}=\lambda_E\lambda_l\end{cases} \tag{12}$$

By substituting Formula (12) into Formulas (9) and (10), the similarity ratio of vibration displacement can be expressed as follows:

$$\lambda_U=\frac{\boldsymbol{U}'}{\boldsymbol{U}}=\frac{\lambda_{\boldsymbol{F}}}{\lambda_l} \tag{13}$$

When the shell is immersed in water, it is subjected to the reaction of the fluid during vibration. In such a case, the loading force of the fluid is produced on the surface of the shell, affecting the vibration characteristics of the shell. The finite element equation of the underwater shell under excited vibration [10] is as follows:

$$\left[\boldsymbol{K}+\mathrm{i}\omega\boldsymbol{C}+\bar{\boldsymbol{R}}-\omega^2\left(\boldsymbol{M}+\bar{\boldsymbol{M}}\right)\right]\boldsymbol{U}=\boldsymbol{F} \tag{14}$$

With the analysis method similar to that used by the scale model test in air, the similarity ratio of vibration responses of the underwater shell can be expressed as follows:

$$\lambda_{U_{\mathrm{f}}}=\frac{\boldsymbol{U}'}{\boldsymbol{U}}=\frac{\lambda_{\boldsymbol{F}}}{\lambda_l} \tag{15}$$

2.3 Similarity of radiated acoustic field

The finite element equation of the fluid-solid interaction of the prototype [10] can be expressed as

$$\boldsymbol{M}_{\mathrm{f}}\omega^2\left[\left(\frac{\omega_{\mathrm{f}}^2}{\omega}-1\right)+\mathrm{i}\frac{\omega_{\mathrm{f}}}{\omega}\eta_{\mathrm{f}}\right]p\omega^2\boldsymbol{SU}=0 \tag{16}$$

where $\boldsymbol{M}_{\mathrm{f}}$ is a mass matrix of the fluid; p is the pressure of the fluid; η_{f} is a loss factor of the fluid; ω_{f} is the underwater natural frequency.

On the basis of Formula (2), the similarity ratio of the coupling matrix of the scale model to that of the prototype can be deduced by calculation and expressed as follows:

$$\lambda_{\mathrm{s}}=\lambda_l^2\lambda_{\rho_{\mathrm{a}}} \tag{17}$$

Where $\lambda_{\rho_{\mathrm{a}}}$ is the ratio of the fluid density of the scale model to that of the prototype.

With the similarity analysis method, by substituting Formula (17) into Formula (16) and comparing expressions of the scale model and the prototype, the paper obtains the similarity ratio of acoustic pressure, as shown in Formula (18).

$$\lambda_{p_{\mathrm{a}}}=\frac{p_{\mathrm{a}}'}{p_{\mathrm{a}}}=\frac{\lambda_U}{\lambda_{r_0}} \tag{18}$$

where p_{a} is the underwater acoustic pressure; λ_{r_0} is the similarity ratio of measuring point distance r_0 in space.

By substituting Formula (15) into Formula (18), the paper obtains the similarity ratio of the acoustic pressure as follows:

$$\lambda_{p_{\mathrm{a}}}=\frac{p_{\mathrm{a}}'}{p_{\mathrm{a}}}=\frac{\lambda_U}{\lambda_{r_0}}=\frac{\lambda_{\boldsymbol{F}}}{\lambda_{r_0}\lambda_l} \tag{19}$$

According to the definition of radiation efficiency, the following formula is deduced.

$$\sigma = \frac{P_{\rm w}}{\rho_{\rm a} c_{\rm a} S V^2} \tag{20}$$

where $c_{\rm a}$ is the underwater acoustic velocity; S is the structural surface area; V is the average vibration velocity of the structural surface; $P_{\rm w}$ is the radiated acoustic power, which can be expressed as

$$P_{\rm w} = \iint p_{\rm a} V^* {\rm d}S \tag{21}$$

Then the similarity ratio of radiation efficiency between the scale model and the prototype is given by

$$\lambda_\sigma = 1 \tag{22}$$

From the above formula, it is known that when the scale model and the prototype are geometrically similar, their boundary conditions are the same, and have the same radiation efficiency.

The far-field acoustic pressure [2] is expressed as follows:

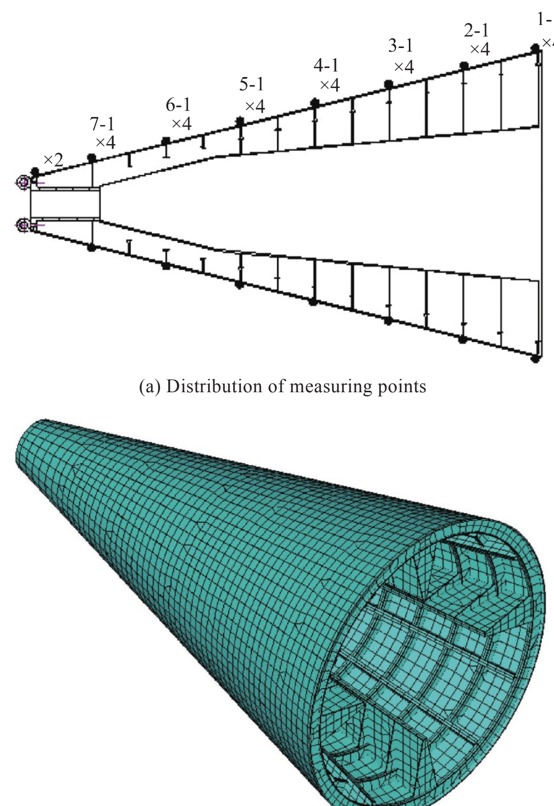
$$P(r,\theta,\psi) = \sum_{m}^{\infty} \sum_{n=m}^{-m} p_{mn(r_0)} \left[\frac{h_m^1(kr)}{h_m^1(kr_0)} \right] Y_m(\theta,\psi) \tag{23}$$

In the formula, $p_{mn(r_0)}$ is the acoustic pressure, where m is the modal order number; n is the serial number of axial wavenumber solutions to the dispersion equation; $h_m^1(kr)$ and $h_m^1(kr_0)$ are the Hankel function of the first kind, where k is a coefficient; Y_m (θ, ψ) is a Legendre function (θ and ψ are azimuth angles).

As can be seen from the above, the far-field acoustic pressure of the scale model and the prototype can be expressed in the same way when they are completely similar. If kr equals $k'r'$, and kr_0 equals $k'r'_0$, the similarity conditions of far-field acoustic pressure are independent of azimuth angles, and thus the acoustic pressure directivity of the scale model and the prototype is the same.

3 Test verification of stiffened conical shell model

Through the vibration response test of a stiffened conical shell model, the structural vibration responses of the model in the air and water were obtained respectively. During the test, with its openings at both ends unclosed, the model was immersed in a water tank through flexible ropes, which made both the inner and outer sides of the model immersed in the fluid. Fig. 2 (a) shows the layout of the model and measuring points of an accelerometer. Specifically, from 1-1 to 7-1, each group contained four measuring points evenly distributed around the model. In addition, there were two measuring points at the top of the conical shell, and they were arranged in horizontal and vertical directions. Fig. 2 (b) shows finite element models of the conical shell built by using beam and shell elements for reinforcement simulation in ANSYS, respectively. At first, the structural vibration response in the air was calculated by mode superposition. Then, the finite element models and modal data were imported into Virtual. Lab Acoustics to calculate the structural vibration response of the underwater model by FEM-BEM. In the simulation and calculation, $\rho_{\rm a}$ was set to 1 000 kg/m^3, and $c_{\rm a}$ was set to 1 500 m/s. The natural frequency and vibration response of the stiffened conical shell model were calculated and tested in a free state without boundary constraints.



(a) Distribution of measuring points

(b) Finite element model

Fig. 2 Distribution of measuring points and finite element model of stiffened conical shell

The vibration response of the stiffened conical shell model in the air was simulated and calculated. Fig. 3 compares simulation and test results. From the figure, it can be seen that the acceleration levels obtained by the test and the simulation are consistent, with close values. However, the corresponding frequencies of peaks are slightly different. Specifically, the first order peak frequency in the test is 106 Hz; the first order peak frequency of the model using shell elements for reinforcement simulation is 102 Hz, and that of the model using beam elements

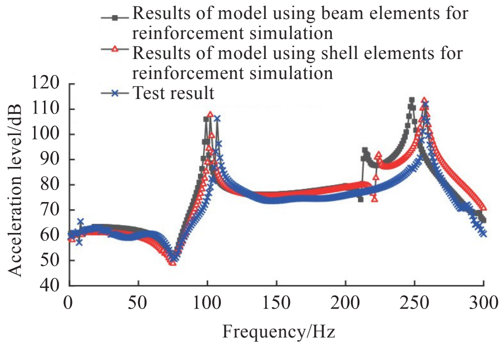


Fig. 3 Comparison of acceleration levels obtained by test and simulation in air

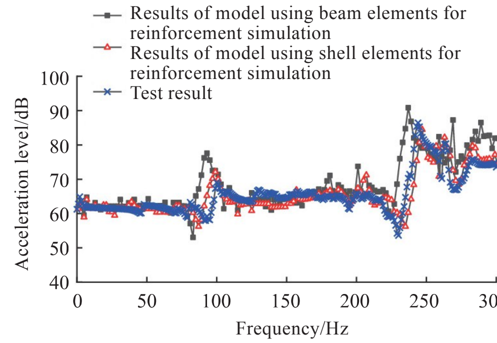


Fig. 4 Comparison of acceleration levels obtained by test and simulation in water

for reinforcement simulation is 99 Hz.

Although the vibration responses obtained by the test and the simulation are generally consistent, the vibration response of the model using shell elements for reinforcement simulation is in better agreement with the test results. This is mainly because the model constructed in this way can simulate the real connection of ring-rib and longitudinal-rib webs, while the model using beam elements for reinforcement simulation fails to do so.

During the calculation of structural vibration responses under fluid coupling in Virtual.Lab Acoustics, the models with the two reinforcement simulation methods were calculated. Then, the calculation and test results were compared to verify the rationality of the two models in calculating the vibration response of the underwater reinforcement structure.

Fig. 4 compares acceleration levels of the vibration response obtained by the test and the simulation of the underwater model constructed in two ways. From Fig. 4, it can be seen that the calculated results are basically consistent with test ones, but the vibration responses at some frequencies are significantly different. Specifically, the vibration response of the underwater model using beam elements for reinforcement simulation has greater amplitude deviations, while that of the model using shell elements for reinforcement simulation agrees better with test results. This is mainly because the model using beam elements for reinforcement simulation fails to realize the coupling of the ring and longitudinal ribs with the fluid in calculating underwater structural vibration responses by BEM, which indicates that it is more reasonable to use shell elements to simulate stiffened structures in fluids when the fluid coupling is considered.

By comparing vibration responses obtained by test and simulation of stiffened conical shell models both in the air and water, the paper finds that the calculated results of the model using shell elements for reinforcement simulation are more consistent with the test ones. This is mainly because the model constructed in this way can better simulate the connection of ring-rib and longitudinal-rib webs. When stiffened structures are immersed in fluids, the structure-fluid coupling effect can be greatly realized by reinforcement simulation with shell elements. Through comparison, it is also found that the vibration response of the underwater conical shell calculated by the FEM-BEM is basically consistent with test results, which indicates that it is feasible to use the FEM-BEM to calculate the vibration responses of underwater shells.

4 Calculation and analysis of similarity law of complex stiffened cone-cylinder shell

Considering that simulation results of the conical model using shell elements for reinforcement simulation are more consistent with test ones, this paper adopted this reinforcement simulation method to construct a complex stiffened cone-cylinder shell model for calculation. According to the finite element theory of acoustic vibration similarity, the paper, based on ANSYS and Virtual.Lab Acoustics, studied acoustic vibration similarity laws of the combined shell by the FEM-BEM.

Firstly, the dry modal of the combined shell in the air was calculated in ANSYS, and its vibration response was calculated by mode superposition. Then, the finite element model constructed in ANSYS and modal data were imported into Virtual. Lab Acoustics to calculate the wet modal, vibration response, and acoustic radiation of the underwater combined shell by BEM. Specifically, the combined shell model was unconstrained. Considering the actual situation, the inner and outer sides of the cone and the outer side of the cylinder were immersed in the fluid, while the inner side of the cylinder was not. During the calculation of underwater acoustic vibration response, the acoustic pressure on the inner side of a boundary-element model grid of the cylinder was set to zero, so as to ensure that the inner side of the cylinder was not immersed in the fluid.

4.1 Natural frequency similarity of stiffened cone-cylinder shell

The modals of the prototype of the complex stiffened cone-cylinder shell and its 1∶4 scale model in the air and water were calculated. Natural frequencies except those of rigid modal in the first 20 orders modal frequencies were extracted. Calculated results are listed in Tables 1 and 2.

According to Table 1, the similarity ratios of modal frequencies between the prototype of the combined shell and its 1∶4 scale model in the air are all about 4, which shows positive similarity law and conforms to the law that modal frequencies are inversely proportional to geometric sizes.

When the combined shell is immersed in water, its natural frequency becomes lower under fluid coupling. By importing the calculated dry modal into Virtual.Lab Acoustics and using BEM to couple fluid action, this paper calculated the wet modal of the combined shell. From Table 2, it can be seen that the similarity ratios of modal frequencies between the prototype of the combined shell and its 1∶4 scale model are all about 4. By comparative analysis of the natural frequency, the paper finds that the similarity ratios of modal frequencies of the combined shell both in the air and water are inversely proportional to geometric sizes.

Table 1 Dry modal frequency of stiffened cone-cylinder shell

Order	Frequency of prototype/Hz	Frequency of scale model/Hz	Ratio	Order	Frequency of prototype/Hz	Frequency of scale model/Hz	Ratio
1	8.1769	32.840	4.02	8	37.943	151.81	4.00
2	17.115	69.110	4.04	9	39.324	162.42	4.13
3	17.180	69.355	4.04	10	39.446	163.10	4.13
4	24.460	100.34	4.10	11	51.852	207.72	4.01
5	30.703	125.35	4.08	12	52.672	211.19	4.01
6	33.470	133.94	4.00	13	52.715	211.39	4.01
7	37.721	150.92	4.00	14	52.753	211.47	4.01

Table 2 Wet modal frequency of stiffened cone-cylinder shell

Order	Frequency of prototype/Hz	Frequency of scale model/Hz	Ratio	Order	Frequency of prototype/Hz	Frequency of scale model/Hz	Ratio
1	4.016	16.025	3.99	8	25.833	109.080	4.22
2	8.816	35.714	4.05	9	27.128	112.514	4.14
3	10.911	45.735	4.19	10	29.214	114.062	3.91
4	15.814	66.846	4.23	11	31.724	124.208	3.92
5	18.601	78.223	4.21	12	33.222	140.557	4.23
6	21.596	88.504	4.10	13	34.035	141.900	4.17
7	24.260	95.861	3.95	14	36.500	147.535	4.04

4.2 Vibration response similarity of stiffened cone-cylinder shell

The model of the complex stiffened cone-cylinder shell has the same exciting points and measuring points of the vibration response both in the air and water. Specifically, a unit exciting force was applied at the coordinate point (0, 0, 1) of the prototype along the x direction, and the vibration response of the prototype was measured at the coordinate point (8, 0, 3.5) at the cone-cylinder junction. The exciting points and measuring points of the vibration response of the scale model correspond to those of the prototype. The vibration response of the combined shell in the air was calculated by mode superposition in ANSYS. The calculated frequency of the prototype ranges from 10 to 100 Hz, with an interval of 1 Hz, while that of the 1∶4 scale model ranges from 40 to 400 Hz, with an interval of 4 Hz. Fig. 5 compares the response curves of vibration acceleration levels of the prototype and the scale model of the complex stiffened cone-cylinder shell in the air. Fig. 6 compares response curves of vibration acceleration levels of the two under fluid coupling.

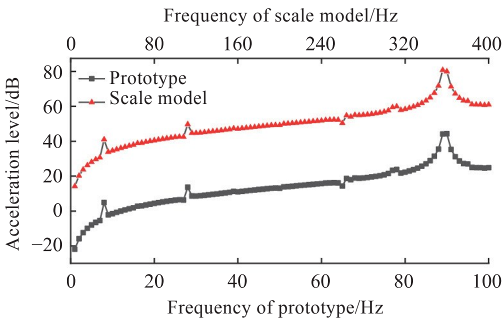


Fig. 5 Comparison of vibration acceleration level response of stiffened cone-cylinder shell in the air

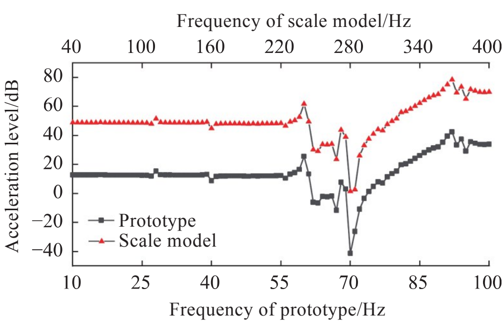


Fig. 6 Comparison of vibration acceleration level response of stiffened cone-cylinder shell in water

From Fig. 5, in the case of the same position of exciting force and point and completely similar structure size, the vibration responses of the scale model and the prototype are consistent, with similar vibration acceleration levels at corresponding frequencies. Moreover, the vibration response of the model is inversely proportional to the geometric scale ratio, which satisfies the similarity ratio of vibration response in the air expressed by Formula (13).

According to Fig. 6, when the shell is immersed in water, fluid coupling affects underwater structural vibration responses obviously. Compared with those in the air, the curves of structural vibration response in water are generally consistent, with denser peaks. This is mainly because the mass effect of the added water leads to a decrease in the structural natural frequency. Despite the effect of fluid coupling, the response curves of vibration acceleration levels of the scale model and the prototype still tend to be consistent, which indicates that the underwater model is of good vibration similarity under excited vibration. In addition, the vibration responses of the scale model and the prototype are approximately inversely proportional to the geometric scale ratio, which conforms to the similarity law of vibration response in water expressed by Formula (15).

4.3 Similarity of radiated acoustic field of stiffened cone-cylinder shell

The underwater acoustic radiation of stiffened cone-cylinder shell models was calculated. Vibration displacement of the model under fluid coupling was applied to boundary-element models to calculate radiated noise in an infinite flow field. The position, direction, and magnitude of the exciting force on both the scale model and the prototype are the same. The radiated acoustic pressure of field points at corresponding positions of the prototype and the scale model was analyzed. Specifically, the field point of the prototype locates at coordinate point (6, 0, 15), and the corresponding field point of the scale model locates at coordinate point (1.5, 0, 3.75). Fig. 7 illustrates underwater radiated acoustic pressure levels at corresponding field points of the stiffened cone-cylinder shell model. From the figure, the curves of radiated acoustic pressure levels at corresponding field points of the prototype and the scale model are basically consistent. As radiated acoustic fields are affected by many factors, the acoustic pressure at some frequencies deviates to some extent. However, the radiated acoustic pressure is still similar on the whole.

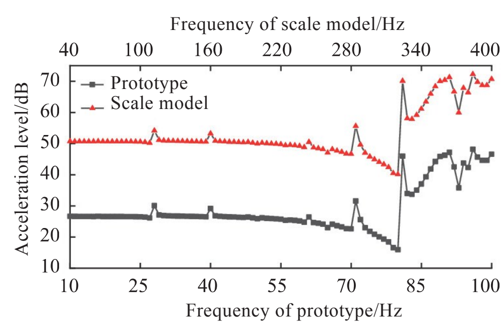


Fig. 7 Comparison of radiated sound pressure of underwater stiffened cone-cylinder shells in field points

The frequency and radiated acoustic pressure at corresponding field points in the acoustic field of the scale model and the prototype were extracted. Table 3 lists frequencies with a frequency ratio of 4 and similarity ratios of corresponding radiated acoustic pressure. According to Formula (19), when both the length ratio and the measuring point distance ratio of the model are 4, the similarity ratio of acoustic radiation is 16. The ratios of radiated acoustic pressure at corresponding points shown in Table 3 are all about 16.

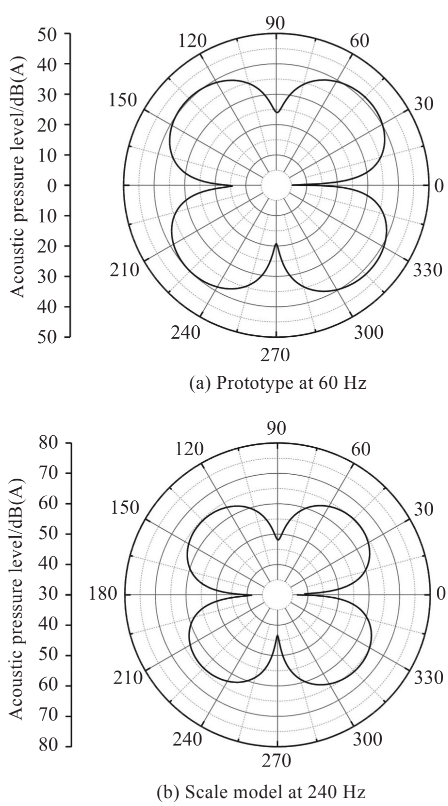
From Table 3 and Fig. 7, the frequency response curves of acoustic pressure of the scale model and the prototype are generally consistent in correspond-

Table 3 Similarity relation of radiated sound pressure

Frequency of prototype/Hz	Frequency of scale model/Hz	Radiated acoustic pressure of prototype/(N·m^{-2})	Radiated acoustic pressure of scale model/(N·m^{-2})	Ratio
11	44	−0.000 496	−0.008	16.1
36	144	0.000 251 8	0.004	15.9
90	360	0.004	0.064	16

ing frequencies. This indicates that the radiated acoustic pressure is of similarity and satisfies the law that acoustic pressure is inversely proportional to the product of the scale ratio and the field point distance ratio expressed by Formula (19).

Fig. 8 illustrates the acoustic pressure directivity at corresponding positions of the underwater stiffened cone-cylinder shell and its scale model. Fig. 8 (a) shows the acoustic pressure directivity of the prototype at 60 Hz, while Fig. 8 (b) illustrates that of the scale model at 240 Hz. Specifically, the directivity radius of the prototype is 60 m, while that of the scale model is 15 m, and both directivity patterns are located on the *xoy* plane. According to the comparison between the underwater combined shell model and its scale model in Fig. 8, with the similar geometry, the same material and fluid properties, and the same loading and excitation positions, the acoustic pressure directivity of the underwater scale model is basically the same as that of the prototype.



(a) Prototype at 60 Hz

(b) Scale model at 240 Hz

Fig. 8 Comparison of acoustic pressure directivity of underwater stiffened cone-cylinder shell

5 Conclusions

In this paper, a complex stiffened cone-cylinder shell model and its scale model were constructed by using shell elements for reinforcement simulation. The FEM-BEM was employed to establish vibration finite element equations of the combined shell model in the air and water, as well as acoustic finite element equations of the model in fluids. On this basis, the paper derived similarity ratios between the combined shell and its scale model in terms of vibration, acoustic-solid coupling vibration, and underwater acoustic fields. Moreover, through the model test, this paper verified the accuracy of using the FEM-BEM to solve the acoustic vibration and studied acoustic-vibration similarity laws of the complex stiffened cone-cylinder shell. The main conclusions are as follows:

1) During the simulation and calculation of acoustic vibration of underwater shells, if the coupling between stiffened structures and flow fields exists, it is more reasonable to use shell elements rather than beam ones for reinforcement simulation.

2) For the complex stiffened cone-cylinder shell, with the same material properties, boundary conditions, and exciting force, its modal frequency is inversely proportional to the geometric scale ratio, and the vibration modal at corresponding frequencies is the same. Moreover, with the same exciting force, the vibration response of the complex combined shell is inversely proportional to the geometric scale ratio, while the acoustic pressure is inversely proportional to the product of the geometric scale ratio and the field point distance ratio.

3) The radiation efficiency and acoustic pressure directivity of the scale model and the prototype are the same.

The research results in this paper can provide support and basis for the scale model based acoustic vibration test of underwater complex stiffened shells.

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复杂加筋的锥—柱组合壳声振相似规律研究

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摘　要:[**目的**] 旨在解决复杂加筋组合壳结构缩尺模型的声振试验结果难以准确换算至原型的难题,分析两者的声振相似规律,为水下复杂加筋壳结构声振缩尺模型的试验研究提供依据。[**方法**] 使用面单元模拟加筋方法构建复杂锥—柱组合壳模型及其缩尺模型,基于有限元—边界元法(FEM-BEM)混合方法,计算模型壳体的声振响应,并结合加筋圆锥壳模型试验,验证采用上述方法计算复杂组合壳声振响应的准确性,系统研究复杂组合壳的声振相似规律。[**结果**] 结果表明:在相同的模型材料参数、边界条件和激励力下,复杂组合壳的模态频率与缩尺比成反比,对应频率的模态振型相同;在相同的激励力下,复杂组合壳的振动响应与缩尺比成反比,其声压与缩尺比和场点距离的乘积成反比;缩尺模型与原型的水下声辐射效率及声压指向性相同。[**结论**] 在模型相似条件下,复杂组合壳结构表现出了良好的声振相似规律,满足相似条件下采用缩尺模型试验研究组合壳原型结构的声振特性要求。

关键词: 锥—柱组合壳; 相似律; 有限元—边界元法混合方法